CE-1
August 2012 QE

Questions 1 and 2 are worth 10 points correct, but -10 points if incorrect. They are worth zero unanswered. No explanations will be considered with answers to questions 1 and 2. The CE-1 overall score cannot be negative, however.

1. (Answer True or False only) Given an undirected graph (V, E) with distinct costs $c(u, v)$ on each edge (no ties between edge costs) and a set $S \subseteq V$, let (u, v) be the lowest cost edge with $u \in S$ and $v \notin S$. Then every minimum cost spanning tree must contain the edge (u, v).

2. (Answer True or False only) If a problem L_1 is polynomial-time reducible to a problem L_2 and L_2 has a polynomial-time algorithm, then L_1 has a polynomial-time algorithm.

3. **(30 points)** Describe an algorithm to insert line breaks into a sequence of words to minimize the square of the number of spaces at the end of each line. You are given a sequence of word-lengths $L_1, \ldots, L_n$ for n words that are to be printed in a fixed-width font (left to right, top to bottom) in a rectangular window that has a width of W characters. One space is printed between each word and the next word on the same line, but no spaces are needed at the end of the line unless the width W of that line has not been reached before the inserted line break. The words are left-aligned and cannot be split between lines. If k words are printed on a line, there will be $k - 1$ spaces printed between them, one space between each consecutive pair of words, along with the characters of the words; any remaining space (up to the width W) is taken by spaces at the end of the line. You are to output an optimal sequence of numbers describing the number of words to be printed on each line; e.g., 5, 4, 4 would mean print the first 5 words on line 1, then the next 4 words on line 2, etc. The output sequence is optimal when it minimizes the sum over all lines used of the square of the number of spaces needed at the end of the line to reach width W. *Carefully and completely describe in English a polynomial-time algorithm for this problem and argue for its correctness and efficiency, OR argue carefully that there is no such algorithm.*

4. **(30 points)** Professor Gafizzit is very worried about cheating on his exams. Although he doesn't understand Facebook, he has become concerned that any pair of students that are Facebook friends will be able to cheat during his exam. (He is not quite sure how.) As a result of his paranoia, he has decided to remove a subset of students from his exam room and give them the exam in a small highly supervised overflow room. He wishes to select the removed students so that no remaining pair of students in the main room are Facebook friends. He has tasked his TA (you) with the problem of selecting at most k students from the class of n students to send to the overflow room. He has (somehow) provided you with the complete Facebook friend data for the class as your input. *Carefully and completely describe in English a polynomial-time algorithm for this problem and argue for its correctness and efficiency, OR argue carefully that there is no such algorithm.*

5. (20 points) Give and prove tight asymptotic bounds for both $S(n)$ and $T(n)$ in the following joint recurrences, assuming $T(1) = \Theta(1)$ and $S(1) = \Theta(1)$:

$$T(n) = 9S(n/3) + \Theta(n^2)$$
$$S(n) = 4T(n/2) + \Theta(n^2)$$

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