Problem 1.(50pt)
Consider the following discrete space system with input $x(m,n)$ and output $y(m,n)$.

$$y(m,n) = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} x(m-k, n-l)h(k,l) \ .$$

For parts a) and b) let

$$h(m,n) = \text{sinc}(mT, nT) \ ,$$

where $T \leq 1$.

a) Calculate the frequency response, $H(e^{j\mu}, e^{j\nu})$.

b) Sketch the frequency response for $|\mu| < 2\pi$ and $|\nu| < 2\pi$ when $T = 1/2$.

For parts c), d), and e) let

$$h(m,n) = \text{sinc}\left(\frac{(n+m)T}{\sqrt{2}}, \frac{(n-m)T}{\sqrt{2}}\right) \ .$$

where $T \leq 1$.

c) Calculate the frequency response, $H(e^{j\mu}, e^{j\nu})$.

d) Sketch the frequency response for $|\mu| < 2\pi$ and $|\nu| < 2\pi$ when $T = 1/2$.

e) Calculate $y(m,n)$ when $x(m,n) = 1$.

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Problem 2.(50pt)
Consider an image $f(x,y)$ with a forward projection

$$\begin{aligned} p_\theta(r) &= \mathcal{FP}\left\{f(x,y)\right\} \\ &= \int_{-\infty}^{\infty} f(r\cos(\theta) - z\sin(\theta), r\sin(\theta) + z\cos(\theta))\, dz \ . \end{aligned}$$

Let $F(u,v)$ be the continuous-space Fourier transform of $f(x,y)$ given by

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}dxdy$$

and let $P_\theta(\rho)$ be the continuous-time Fourier transform of $p_\theta(r)$ given by

$$P_\theta(\rho) = \int_{-\infty}^{\infty} p_\theta(r)e^{-j2\pi(\rho r)}dr \ .$$

a) Calculate the forward projection $p_\theta(r)$, for $f(x,y) = \delta(x,y)$.

b) Calculate the forward projection $p_\theta(r)$, for $f(x,y) = \delta(x-1, y-1)$.

c) Calculate the forward projection $p_\theta(r)$, for $f(x,y) = \text{rect}\left(\sqrt{x^2+y^2}\right)$.

d) Calculate the forward projection $p_\theta(r)$, for $f(x,y) = \text{rect}\left(\sqrt{(x-1)^2+(y-1)^2}\right)$.

d) Describe in precise detail, the steps required to perform filtered back projection (FBP) reconstruction of $f(x,y)$.

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