CS-4
August 2011 QE

1. **(25 points)** State whether the following statements are true or false. **No justification is necessary.**

   (a) (5 points) For a $G/G/1$ queue with arrival rate λ and service rate $\mu > \lambda$. The fraction of time that the system is empty is always equal to $1 - \lambda/\mu$, regardless of the arrival pattern and the service time distribution.

   (b) (5 points) One of the design choices of the Internet is to provide best-effort service in the IP protocol, so that quality-of-service such as real-time delivery guarantees can be provided in the high-layer TCP protocol.

   (c) (5 points) A closed queueing network can be unstable when the arrival rate to the system is very large.

   (d) (5 points) As the packet size increases, the throughput of Go-Back-N ARQ protocol approaches that of Selective Repeat.

   (e) (5 points) The Dijkstra's algorithm can compute the minimum-cost path for any graph, even when some of the costs are negative.

2. **(30 points)** Consider a random access system with two channels. Two packets can be transmitted simultaneously in the system, as long as they are transmitted on different channels. In this problem, you will be asked to derive the throughput of pure-Aloha for such a two-channel system. Assume that there are N stations. New packets arrive at each station according to a Poisson process with rate λ, independently of other stations. Assume that each packet is of fixed length. However, the two channels have different transmission rates, and hence the transmission times for a packet are different on the two channels. Specifically, let m_1 denote the transmission time of a packet on channel 1, and let m_2 denote the transmission time of a packet on channel 2. When a station has a new packet to send, it will randomly choose channel 1 or channel 2 with probability p and $1-p$, respectively, and immediately transmit the packet on the chosen channel. If there is a collision on the chosen channel, each station involved in the collision will then wait for a random time, again randomly and independently choose channel 1 or channel

2 with probability p and $1-p$, respectively, and retransmit the packet on the chosen channel. Such a procedure continues until the packet is successfully transmitted.

Let λ' denote the aggregate rate of packet transmissions at each station (including both new and retransmitted packets). You may assume that the aggregated arrivals of both new and retransmitted packets on each channel are Poisson.

(a) (10 points) For a given packet transmission on channel 1 that starts at time t (which can be either a new packet or a retransmitted packet), find the probability that this packet can be transmitted successfully (i.e., with no collision).

(b) (10 points) Derive an expression that relates λ and λ'. (Note: packet transmissions may happen on either channel 1 or channel 2.)

(c) (10 points) Assume that $p = m_2/(m_1 + m_2)$, i.e., a station is more likely to pick a channel with smaller transmission time. Determine the maximum possible λ that can be supported by this system.

3. (**45 points**) Jobs arrive to a two-queue system in Fig. 1 according to a Poisson process with rate λ. An inspector will inspect queue 1 at times that follow a Poisson process with rate $\mu > \lambda$. If there are any jobs waiting in queue 1 at the time when the inspector inspects queue 1, he will serve the head-of-line job waiting in queue 1 (i.e., first-come first-serve). Otherwise, if queue 1 is empty, the inspector leaves immediately. Similarly, an inspector will inspect and serve jobs in queue 2 at times that follow a Poisson process with rate μ. Assume that both queues have infinite waiting rooms, and assume that the time for the inspector to serve a job is negligible. In parts (a) to (c), assume that the inspection time-instants for queue 1 are independent of that for queue 2.

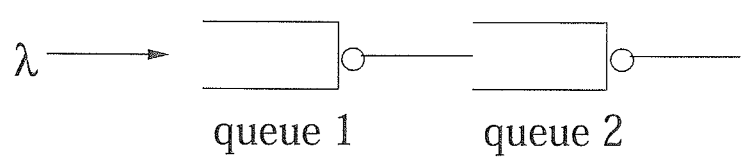

Figure 1: A two-queue system

Write in Exam Book Only

(a) (10 points) Suppose that a job arrives to queue 1 and it is the only job in queue 1. Let T be the amount of time before the job can be served. Derive the distribution of T.

(b) (15 points) Draw the state-transition diagram of the system and derive the steady-state probability that there are n jobs in queue 1.

(c) (10 points) Find the expected number of jobs waiting in the system (queue 1 plus queue 2).

(d) (10 points) In contrast to part (a) to (c), now assume that the inspectors for the two queues always come at the same time. In other words, the inspection time-instants of the two queues follow the *same* Poisson process with rate μ. Find the expected delay of a job from the time when it arrives at queue 1 to the time when it completes service at queue 2.