Problem 1. (40 points) Suppose a system is monitored daily, except on Sundays, because it is faulty half of the time. On Mondays, Wednesdays, and Fridays at noon, if everything checks out fine, a sensor sends a signal with amplitude $-A$, where A is a positive constant, millivolts along a pair of wires. However, if there is a fault, the sensor sends a signal with amplitude A millivolts along the pair of wires. In both cases the duration of the signal from the sensor is ten seconds. The signals are flipped on Tuesday, Thursday, and Saturday, that is, the positive polarity indicates that everything is fine and a negative polarity indicates that there is a fault on these days. The additive white Gaussian two-sided noise power spectral density on the wires is $\frac{N_0}{2} = 5 \times 10^{-6}\ V^2/\text{Hz}$.

(a) Precisely define a receiver structure that takes the signal from the pair of wires in order to determine whether a fault exists in the system on Mondays, Wednesdays, and Fridays. The receiver must minimize the probability of being wrong in this fault assessment. If there is any filtering of the signal on the wire, sampling, or comparison with a threshold in your design, precisely define all filters, sampling times, and threshold values.

(b) Explain how the system that you designed in Part (a) should be modified to work on Tuesdays, Thursdays, and Saturdays.

(c) Find the probability that the optimal system-fault-detecting receiver (of Part (a)) is wrong in its assessment of the condition of the system.

(d) Suddenly, the system is fixed and works perfectly, but now the clocked power to the sensor begins to fail so that half of the weeks the sensor works perfectly as described and half of the weeks it sends no signal at all. Precisely define (as before) a receiver structure that begins working at noon on Monday to detect whether the sensor is working or not and that minimizes the probability of being wrong in this assessment. It must report its answer by Sunday.

(e) Find the probability that the optimal sensor-fault-detecting receiver (of Part (d)) is wrong in its assessment of the condition of the sensor.

(f) How does your receiver design change in Part (d) if, because of temperature changes in the system, the two-sided noise power spectral density becomes $10 \times 10^{-6}\ V^2/\text{Hz}$ on Tuesdays, Thursdays, and Saturdays and remains at $5 \times 10^{-6}\ V^2/\text{Hz}$ on Mondays, Wednesdays, and Fridays?

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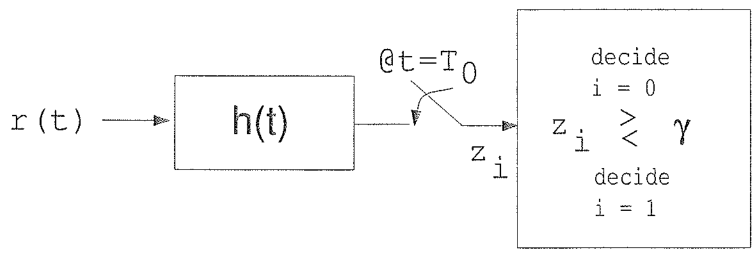
Problem 2. (40 points) (Assume standard units. Your answer should be accurate to three significant figures. When necessary express answers in terms of $\Phi(x)$, the cumulative distribution function of a zero-mean, unit-variance Gaussian random variable.) The real baseband received signal $r(t)$ is given by

$$r(t) = s_i(t) + n(t); i = 0, 1,$$

where $\Pr\{i = 0\} = \Pr\{i = 1\} = 1/2$, and $n(t)$ is an additive white Gaussian noise process with two-sided power spectral density $N_0/2 = 7$. The two real signals are defined by $s_0(t) = e^{-2t}p_{10}(t)$ and $s_1(t) = -e^{-2t}p_{10}(t)$, where $p_{10}(t) = 1$, for $0 \le t < 10$, and $p_{10}(t) = 0$, elsewhere. Suppose that the receiver structure is given as shown below. The received signal is processed with a



linear time-invariant filter with impulse response $h(t)$, and the output of that filter is sampled at time $t = T_0$ to produce the statistic z_i. I If $z_i > \gamma$, i=0 is decided. If $z_i < \gamma$, i=1 is decided.

(a) Find the matched filter for achieving the optimal minimax error probability. (The minimax error probability is defined as the minimum of the maximum of the probability of error given that zero is sent and the probability of error given that one is sent.) Also, specify the sampling time T_0.

(b) Find the optimal minimax error probability.

(c) Find the noise power spectral density at the output of the matched filter.

(d) Find the the total noise power at the output of the matched filter.

(e) Suppose the output of the matched filter is sampled at $t = 3$ seconds to produce a sample $\hat{r}(3)$ and at $t = 3.5$ seconds to produce a sample $\hat{r}(3.5)$. Find the correlation coefficient ρ between the two samples, defined by $\rho = \frac{Cov(\hat{r}(3),\hat{r}(3.5))}{\sqrt{Var\{\hat{r}(3)\}Var\{\hat{r}(3.5)\}}}$, where Cov denotes the covariance and Var denotes the variance.

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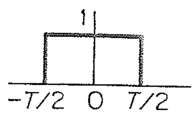
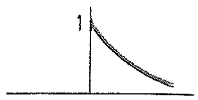
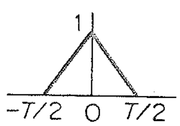
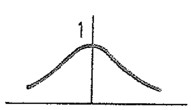
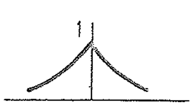
Problem 3. (20 points) In a short paragraph define the pros and cons of using quaternary phase shift keying (QPSK) in place of binary phase shift keying (BPSK):

(1) Describe the pros and cons regarding spectral efficiency.

(2) Describe the pros and cons regarding design of the power amplifier.

(3) Define the pros and cons regarding carrier synchronization.

	$x(t)$	$X(\omega)$
 Rectangular pulse	$u\left(t+\frac{T}{2}\right)-u\left(t-\frac{T}{2}\right)$	$T\,\frac{\sin(\omega T/2)}{\omega T/2}$
 Exponential	$e^{-\alpha t}u(t)$	$\frac{1}{j\omega+\alpha}$
 Triangular	$1-2\frac{\lvert t\rvert}{T},\ \lvert t\rvert<\frac{T}{2}$ 0 elsewhere	$\frac{T}{2}\left[\frac{\sin(\omega T/4)}{\omega T/4}\right]^2$
 Gaussian	$e^{-\alpha^2 t^2}$	$\frac{\sqrt{\pi}}{\alpha}\,e^{-(\omega^2/4\alpha^2)}$
 Double exponential	$e^{-\alpha\lvert t\rvert}$	$\frac{2\alpha}{\alpha^2+\omega^2}$

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