CS-2
August 2011 QE

Cover Sheet

NOTES:

- You need only plot the magnitude of a DTFT (Discrete-Time Fourier Transform) over $-\pi < \omega < \pi$, but it is very important to keep in mind that a DTFT is always periodic with period 2π.
- You must clearly label the DTFT magnitude plot requested and show as much detail as possible, clearly pointing out regions over $-\pi < \omega < \pi$ for which the DTFT is zero.
- You MUST show all work and explain how you got your answer concisely but with sufficient detail to receive full credit.

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Problem 1. [60 pts]

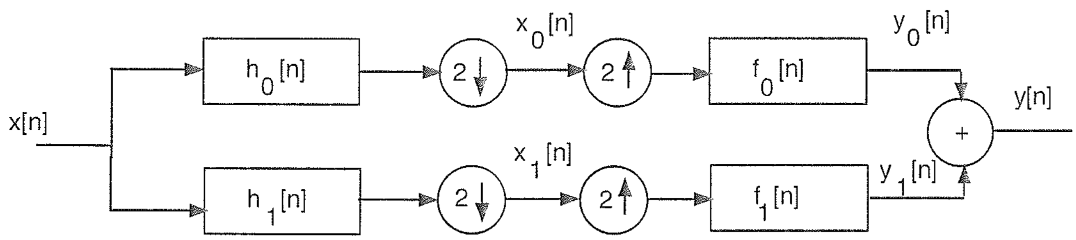
In the system below, the two analysis filters, $h_0[n]$ and $h_1[n]$, and the two synthesis filters, $f_0[n]$ and $f_1[n]$, form a Quadrature Mirror Filter (QMF). Specifically,

$$h_0[n] = \frac{2\beta\cos[(1+\beta)\pi(n+.5)/2]}{\pi[1-4\beta^2(n+.5)^2]} + \frac{\sin[(1-\beta)\pi(n+.5)/2]}{\pi[(n+.5)-4\beta^2(n+.5)^3]}, \quad -\infty < n < \infty \text{ with } \beta = 0.5 \tag{1}$$

$$h_1[n] = (-1)^n h_0[n] \qquad f_0[n] = h_0[n] \qquad f_1[n] = -h_1[n]$$

The DTFT of the halfband filter $h_0[n]$ above may be expressed as follows:

$$H_0(\omega) = \begin{cases} e^{j\frac{\omega}{2}}, & |\omega| < \frac{\pi}{4} \\ e^{j\frac{\omega}{2}}\cos[(|\omega| - \frac{\pi}{4})], & \frac{\pi}{4} < |\omega| < \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < |\omega| < \pi \end{cases}$$



Consider the following input signal

$$x[n] = 16\frac{\sin\left(\frac{3\pi}{8}n\right)}{\pi n}\frac{\sin\left(\frac{\pi}{8}n\right)}{\pi n}\cos\left(\frac{\pi}{2}n\right)$$

HINT: The solution to problem is greatly simplified if you exploit the fact that the DTFT of the input signal $x[n]$ is such that $X(\omega) = X(\omega - \pi)$.

(a) Plot the magnitude of the DTFT of $x[n]$, $X(\omega)$, over $-\pi < \omega < \pi$. Show all work.

(b) Plot the magnitude of the DTFT of $x_0[n]$, $X_0(\omega)$, over $-\pi < \omega < \pi$. Show all work.

(c) Plot the magnitude of the DTFT of $x_1[n]$, $X_1(\omega)$, over $-\pi < \omega < \pi$. Show all work.

(d) Plot the magnitude of the DTFT of $y_0[n]$, $Y_0(\omega)$, over $-\pi < \omega < \pi$. Show all work.

(e) Plot the magnitude of the DTFT of $y_1[n]$, $Y_1(\omega)$, over $-\pi < \omega < \pi$. Show all work.

(f) Plot the magnitude of the DTFT of the final output $y[n]$, $Y(\omega)$, over $-\pi < \omega < \pi$.

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Problem 2. [40 pts]

(a) Let $x[n]$ and $y[n]$ be real-valued sequences both of which are even-symmetric: $x[n] = x[-n]$ and $y[n] = y[-n]$. Under these conditions, prove that $r_{xy}[\ell] = r_{yx}[\ell]$ for all ℓ.

(b) Express the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n] = x[n] + jy[n]$ where $x[n]$ and $y[n]$ are real-valued sequences, in terms of $r_{xx}[\ell]$, $r_{xy}[\ell]$, $r_{yx}[\ell]$, and $r_{yy}[\ell]$.

(c) Determine a closed-form expression for the autocorrelation sequence $r_{xx}[\ell]$ for the signal $x[n]$ below.

$$x[n] = \left\{ \frac{\sin(\frac{\pi}{4}n)}{\pi n} \right\} \{1 + (-1)^n\} \tag{2}$$

(d) Determine a closed-form expression for the autocorrelation sequence $r_{yy}[\ell]$ for the signal $y[n]$ below.

$$y[n] = \left\{ \frac{\sin(\frac{\pi}{4}n)}{\pi n} \right\} \cos\left(\frac{\pi}{2}n\right) \tag{3}$$

(e) Determine a closed-form expression for the autocorrelation sequence $r_{zz}[\ell]$ for the complex-valued signal $z[n]$ formed with $x[n]$ and $y[n]$ defined above as the real and imaginary parts, respectively, as defined below. *You must show all work and simplify as much as possible.*

$$z[n] = x[n] + jy[n] \tag{4}$$

(f) Plot $r_{zz}[\ell]$.

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