AC-3
August 2011 QE

Warning: Graphical solutions are not acceptable! Only analytical solutions will be graded!

1. **(20 pts)** Consider the optimization problem,

$$\begin{aligned} &\text{maximize} \quad -x_1^2 + x_1 - x_2 - x_1 x_2 \\ &\text{subject to} \quad x_1 \geq 0,\ x_2 \geq 0. \end{aligned}$$

(i) **(10 pts)** Characterize feasible directions at the point

$$\boldsymbol{x}^* = \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix}.$$

(ii) **(10 pts)** Write down the second-order necessary condition for $\boldsymbol{x}^*$. Does the point $\boldsymbol{x}^*$ satisfy this condition?

2. **(20 pts)** Use the simplex method to solve the problem,

$$\begin{aligned} &\text{maximize} \quad x_1 + x_2 \\ &\text{subject to} \quad x_1 - x_2 \leq 2 \\ &\qquad\qquad\quad x_1 + x_2 \leq 6 \\ &\qquad\qquad\quad x_1, x_2 \geq 0. \end{aligned}$$

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3. **(20 pts)** Solve the following linear program,

$$\begin{aligned} \text{maximize} \quad & -x_1 - 3x_2 + 4x_3 \\ \text{subject to} \quad & x_1 + 2x_2 - x_3 = 5 \\ & 2x_1 + 3x_2 - x_3 = 6 \\ & x_1 \text{ free},\ x_2 \geq 0,\ x_3 \leq 0. \end{aligned}$$

4. **(20 pts)** Consider the following model of a discrete-time system,

$$x(k+1) = 2x(k) + u(k), \quad x(0) = 0, \quad 0 \leq k \leq 2$$

Use the Lagrange multiplier approach to calculate the optimal control sequence

$$\{u(0), u(1), u(2)\}$$

that transfers the initial state $x(0)$ to $x(3) = 7$ while minimizing the performance index

$$J = \frac{1}{2} \sum_{k=0}^{2} u(k)^2$$

5. **(20 pts)** Consider the following optimization problem,

$$\begin{aligned} \text{optimize} \quad & (x_1 - 2)^2 + (x_2 - 1)^2 \\ \text{subject to} \quad & x_2 - x_1^2 \geq 0 \\ & 2 - x_1 - x_2 \geq 0 \\ & x_1 \geq 0. \end{aligned}$$

The point $\boldsymbol{x}^* = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ satisfies the KKT conditions.

(i) **(10 pts)** Does $\boldsymbol{x}^*$ satisfy the FONC for minimum or maximum? What are the KKT multipliers?

(ii) **(10 pts)** Does $\boldsymbol{x}^*$ satisfy the SOSC? Carefully justify your answer.

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