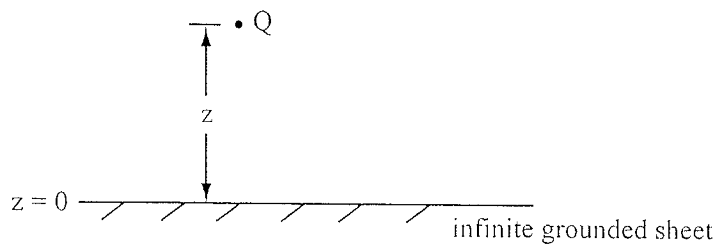
FO-1
August 2007

1. (40 pts.)

(a) An infinite grounded conducting plane has its top surface at $z = 0$. Determine the work required to move a point charge Q from $z \to \infty$ to $z = z_f$.

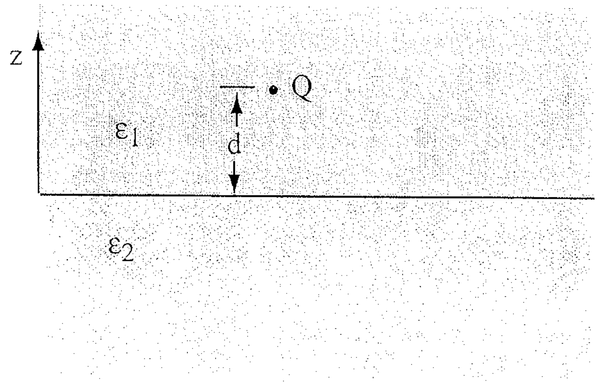


(b) Determine the electrostatic force on a charge Q located a distance z from an infinite grounded conducting plane. Is it attractive or repulsive? Explain.

(c) We know that the work we do in moving an object along a path from point A to point B is $W = \int_A^B \vec{F} \cdot d\vec{r}$, where $\vec{F}$ is the force we apply to the body as it is displaced $d\vec{r}$ along the path. Compare the results of parts (a) and (b). Discuss any differences or similarities.

2. (30 pts.)

A point charge Q is suspended in a dielectric material of permittivity ε_1, a distance d above the planar interface with a second dielectric materials of permitivity ε_2, as shown.

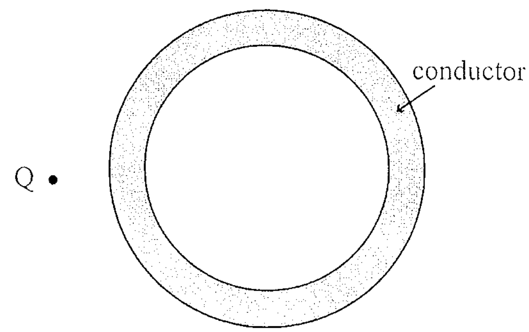


(a) For $\varepsilon_2 > \varepsilon_1$, sketch the electric field lines $\vec{E}$. (No derivations are necessary for your sketch. You will need to redraw the figure above in your answer book.) How do the $\vec{E}$ field lines behave at the interface? Explain in terms of boundary conditions that must be satisfied.

(b) Does the charge experience an electrostatic force? If so, in which direction? Using language understandable to an undergraduate ECE or Physics student, discuss the reasoning behind this force.

3. (30 pts.)

A point charge Q is brought into the neighborhood of a hollow, spherical, electrically neutral conductor. Indicate on the diagram (You will need to redraw the figure in your answer book.) the distribution of all charges induced on the surfaces (outer and inner) of the conductor. Discuss/explain your answer. No derivations are required for your response.



Maxwell's Equations, in differential and integral form are: Other potentially useful relationships are:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \oint_C \vec{E} \cdot d\vec{\ell} = -\frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{s} \qquad \vec{D} = \varepsilon_o \vec{E} + \vec{P} = \varepsilon_o \vec{E}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \oint_C \vec{H} \cdot d\vec{\ell} = \int_S \left(\vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s} \qquad \vec{B} = \mu_o \left(\vec{H} + \vec{M} \right) = \mu \vec{H}$$

$$\vec{\nabla} \cdot \vec{D} = \rho \qquad \oint_S \vec{D} \cdot d\vec{s} = Q \qquad \lambda = \frac{2\pi}{\beta} = \frac{u}{f} = \frac{1}{f\sqrt{\mu\varepsilon}}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \qquad \oint_S \vec{B} \cdot d\vec{s} = 0 \qquad \varepsilon_o = 8.854 \times 10^{-12} \, F/m$$

$$\mu_o = 4\pi \times 10^{-7} \, H/m$$

Vector Differential Relationships in Cylindrical Coordinates (r, φ, z)

$$\vec{\nabla} V = \hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\phi \frac{\partial V}{r \partial \phi} + \hat{a}_z \frac{\partial V}{\partial z}$$

$$\vec{\nabla} \cdot \vec{A} = \frac{1}{r} \frac{\partial}{\partial r} (r A_r) + \frac{\partial A_\phi}{r \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\vec{\nabla} \times \vec{A} = \hat{a}_r \left(\frac{\partial A_z}{r \partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{a}_\phi \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \hat{a}_z \frac{1}{r} \left[\frac{\partial}{\partial r} (r A_\phi) - \frac{\partial A_r}{\partial \phi} \right]$$

$$\nabla^2 V = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Divergence Thm: $\int_V \vec{\nabla} \cdot \vec{A} dv = \oint_S \vec{A} \cdot d\vec{s}$

Stokes Thm: $\int_V \vec{\nabla} \times \vec{A} \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{\ell}$

For lossless transmission lines:

$$Z_{in} = R_o \frac{Z_L + jR_o \tan\beta\ell}{R_o + jZ_L \tan\beta\ell}$$