CS-5
August 2007

Problem 1.(34pt)
Let $h(m,n)$ be a low pass filter with an impulse response of

$$h(m,n) = \begin{cases} 1/9 & |m| \le 1 \text{ and } |n| \le 1 \\ 0 & \text{otherwise} \end{cases} .$$

Then define the filter impulse response

$$g(m,n) = \delta(m,n) + \lambda(\delta(m,n) - h(m,n))$$

where λ is an parameter which can be adjusted to achieve a desired result and $\delta(m,n)$ is a discrete-space impulse.

a)(12pt) Calculate an analytical expression for $H(e^{j\mu}, e^{j\nu})$ the DSFT of $h(m,n)$.

b)(11pt) Calculate an analytical expression for $G(e^{j\mu}, e^{j\nu})$ the DSFT of $g(m,n)$.

c)(11pt) What is the effect of increasing and descreasing λ in the range $\lambda > 0$.

Problem 2.(33pt)

Consider the 2D difference equation

$$y(m,n) = bx(m,n) + ay(m-1,n) + ay(m,n-1) - a^2 y(m-1,n-1)$$

where $b \in \Re$ and $a \in (-1,1)$ are two constants, and $Y(z_1,z_2)$ and $X(z_1,z_2)$ are the 2D Z-transforms of $y(m,n)$ and $x(m,n)$ respectively.

a)(11pt) Calculate $H(z_1,z_2) = \frac{Y(z_1,z_2)}{X(z_1,z_2)}$, the 2D transfer function of the causal system. Make sure to express your result in factored form.

b)(11pt) Calculate, $h(m,n)$, the impulse response of the system with transfer function $H(z_1,z_2)$.

c)(11pt) In an application, $x(m,n)$ is an input image, and $y(m,n)$ is an output filtered image. Specify a relationship between a and b so that the average values of the input and output images remain the same.

Problem 3.(33pt)

Consider an achromatic image, $X(m,n)$, which is gamma corrected with $\gamma = 2.2$ using simple power-law gamma correction.

a)(11pt) What is the formula used to convert $X(m,n)$ to $I(m,n)$, where $I(m,n)$ has units proportional to energy (i.e. number of photons)?

b)(11pt) If it is necessary to represent the image with 8 bit quantization, is it better to quantize $X(m,n)$ or $I(m,n)$? Why?

c)(11pt) If you need to accurately view the image $X(m,n)$ on a display with a gamma of 1.8, what must you do? Be specific.