CS-4
August 2007

1. (**25 points**) State whether the following statements are true or false. No justification is necessary

   (a) (10 points) Let $\{N_i(t), t \geq 0, i = 1, 2, 3, \ldots, K\}$ be K independent Poisson Processes with rate λ. Assume that K is a Poisson random variable independent of $N_i(t)$ (for all i) and has mean a. Let $N(t) = \sum_{i=1}^{K} N_i(t)$.

      i. (5 points) **Statement:** $\{N(t), t \geq 0\}$ is a Poisson process with rate $a\lambda$.

      ii. (5 points) **Statement:** For a given t and T, $(N(t+T) - N(t))$ is a Poisson random variable with mean $a\lambda T$.

   (b) (5 points) **Statement:** In the slow-start phase, TCP sender increases the congestion-window by 1 every round-trip time.

   (c) (5 points) Consider a stationary packet-arrival process (i.e., assuming that it has started for infinite amount of time from the past). Let $X_i, -\infty < i < \infty$ denote the inter-arrival time between the i-th packet and the $(i+1)$-th packet. Assume that for each i, X_i is $i.i.d.$ exponentially distributed. Let t denote an arbitrary time at which the packet stream is observed. Let S denote the amount of time between the time t and the arrival time of the last packet before t.
   Statement: It then follows that

$$E(X_i) = E(S).$$

   (d) (5 points) **Statement:** For a given utilization, the $M/M/1$ queue has the smallest average delay among all $M/G/1$ queueing systems.

2. (45 points) Consider the following single-server queueing system with an infinite buffer. Packets arrive according to a Poisson process with rate λ. The size of each packet is *i.i.d* exponentially distributed with mean $1/\mu$. The server has a capacity of 1. When there is only one packet in the system, the server will serve this packet at the rate of 1. Hence, a single packet with length l will take time l to complete service. However, *whenever* there are two or more packets in the system, the server will split its capacity into two parts, and serve the first two packets in the queue simultaneously, each at the rate of 1/2. For example, if two packets arrive at an empty system at the same time, and both have size l, then they will also complete service at the same time, after a time $2l$.

   Let P_m denote the steady-state probability of having m packets in the queue.

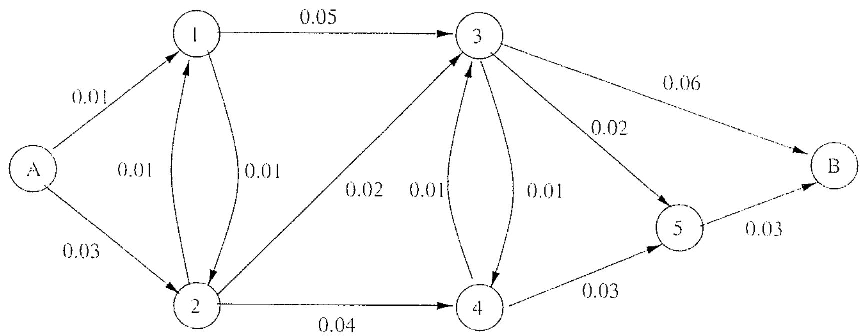
   (a) (15 points) Assume that at a particular time-instant t_0, there are m packets in the queue, and $m \geq 2$. Let $t_0 + T$ denote the first time-instant after t_0 when a packet completes service. Carefully derive the probability $P[T \geq t]$ for all t.

   (b) (10 points) Draw the state transition diagram of the continuous time Markov chain that can be used to determine P_m.

   (c) (10 points) Write down the balance equations for solving P_m.

   (d) (10 points) Compute P_0, the probability that the system is empty.

3. (30 points) Consider the network shown below. The labels on the links correspond to the probabilities of link failure. Assume that the links fail independently of each other. Let node A be the source. Our goal is to find the most reliable path from the source A to every other node.



(a) (15 points) Provide a modified version of Dijkstra's algorithm to find the most reliable path from the source A to each node in the network above.

(b) (15 points) Apply your modified algorithm to the network above to find the most reliable paths from the source A to each node. Show at least three iterations.