1. (35 pts.) A communication system transmits one of three signals:

$$s_0(t) = Kp_T(t)\sin(\omega_c t),$$

$$s_1(t) = 0, \text{ and}$$

$$s_2(t) = -Kp_T(t)\sin(\omega_c t),$$

where K is a positive number and $p_T(t) = 1$ for $0 \le t < T$, and $p_T(t) = 0$, elsewhere.

over an additive white Gaussian noise channel with spectral density $N_0/2$ at a carrier frequency ω_c satisfying $\omega_c \gg 1/T$. Let $W(t)$ denote the received signal ($W(t) = X(t) + s_i(t)$ for i=0,1,2, where $X(t)$ is the noise process). The receiver computes the quantity

$$U = \int_0^T W(t)\sin(\omega_c t)dt.$$

U is compared with a positive threshold γ and a threshold $-\gamma$. If $U > \gamma$, the decision is made that $s_0(t)$ was sent. If $U < -\gamma$, the decision is made that $s_2(t)$ was sent. If $-\gamma < U < \gamma$, the decision is made in favor of $s_1(t)$.

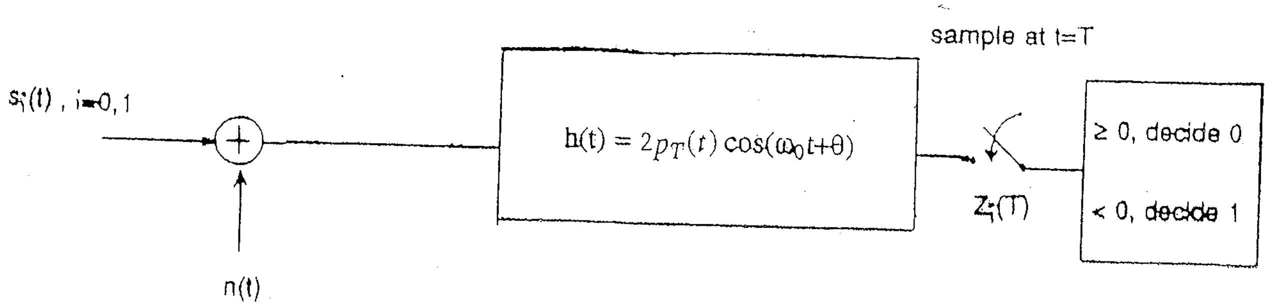
(a) Determine the three conditional probabilities of error: $P_{e,0}$ = probability of error given s_0 sent, $P_{e,1}$ = probability of error given s_1 sent, and $P_{e,2}$ = probability of error given s_2 sent. Express your answer in terms of the Q function defined by $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-u^2/2}du$.

(b) Determine the average error probability assuming that all three signals are transmitted with equal probability.

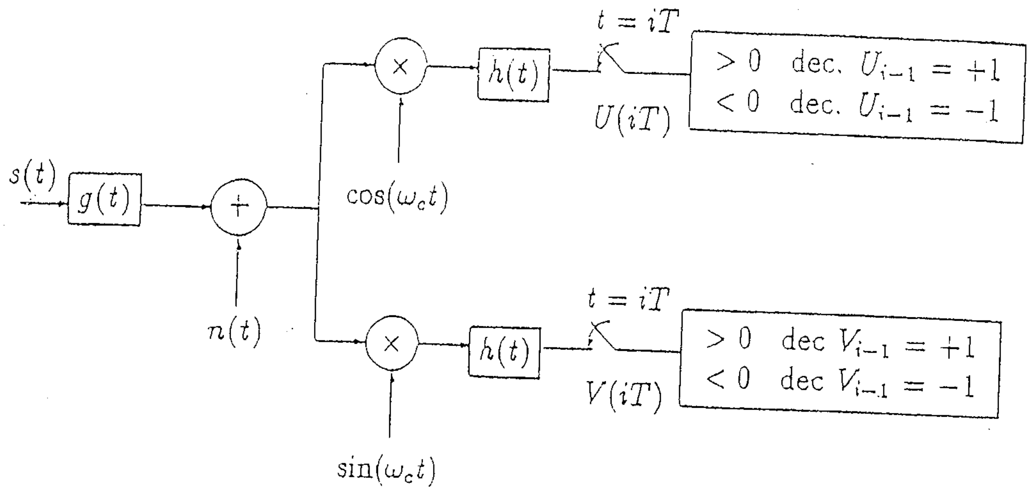
2. (30 pts.) A binary communication system transmits one of two equally likely signals $s_0(t)$ and $s_1(t)$ of duration T given for i=0,1 by

$$s_i(t) = B\,(-1)^i p_T(t)\cos(\omega_0 t).$$

The noise n(t) in the system is white Gaussian noise with power spectral density $N_0/2$. The receiver shown below is used to demodulate the signal. However, as shown, the phase of the received signal is not known completely accurately. In fact, there is a discrepancy of θ radians between the received signal (in the absence of noise) and the local reference signal. Determine the error probability at the output of the demodulator as a function of θ. (Ignore double frequency terms and express your answer in terms of the Q function.)



3\. (35 pts.) The receiver in a QPSK communications system consists of two parallel branches as shown below.



Each branch has a multiplier, a linear time-invariant filter, a sampler, and a threshold device. The baseband impulse response h(t) (with bandwidth much smaller than the carrier frequency ω_c) satisfies

$$\int_{-\infty}^{\infty} h^2(t)dt = 9$$

The channel is an AWGN (additive white Gaussian noise) channel and $\omega_c \gg 1/T$. The white Gaussian noise process $n(t)$ has spectral density $N_0/2 = 2$. The transmitted signal $s(t)$ is given by

$$s(t) = Au(t)\cos \omega_c t + Av(t)\sin \omega_c(t).$$

The impulse response $g(t)$ is unknown. It is known that if $n(t) = 0$, $u(t) = 0$, and $v(t) = p_T(t)$, then

$$V(T) = 5, \quad V(2T) = 2, \quad U(T) = 1,$$

and all other values of $V(kT)$ and $U(nT)$ are zero. Similarly, if $n(t) = 0$, $u(t) = p_T(t)$, and $v(t) = 0$, it is observed that

$$U(T) = 5, \quad U(2T) = 3, \quad V(T) = 1,$$

and all other values of $U(nT)$ and $V(kT)$ are zero.

Give an expression for the average probability of error for each of the two branches of the receiver if $u(t)$ and $v(t)$ are now statistically independent, infinite sequences of unit amplitude, positive and negative, rectangular pulses of duration T. That is,

$$u(t) = \sum_{n=-\infty}^{\infty} U_n p_T(t - nT) \ \ and \ \ v(t) = \sum_{k=-\infty}^{\infty} V_k p_T(t - kT),$$

where $\{U_n\}$ and $\{V_k\}$ are statistically independent sequences of independent random variables with $P(U_n = +1) = P(U_n = -1) = P(V_k = +1) = P(V_k = -1) = 1/2$. Express your answer in terms of the Q function. (Please do not leave this problem blank, but at least calculate the variance of the decision statistics.)