Problem 1. [70 pts] Consider the discrete-time system described by the difference equation below.

$$y[n] = x[n] - x[n-4] \tag{1}$$

(a) (10 pts) Plot the magnitude $|H(e^{j\omega})|$ of the frequency response of this system, equal to the Discrete Time Fourier Transform (DTFT) $H(e^{j\omega})$ of the impulse response $h[n]$, over $-\pi < \omega < \pi$. Show as much detail as possible.

For all parts, a discrete-time (DT) random process $x[n]$ is input to the system above. The DT random process is obtained by sampling a continuous time signal $x_a(t)$ whose autocorrelation function, $r_{x_a x_a}(\tau) = \mathcal{E}\{x_a(t)x_a(t-\tau)\}$. is given, where $\mathcal{E}\{\cdot\}$ represents expected value operator in a statistical sense.

For EACH part below, you must do EACH of the following THREE steps.

(i) Plot the spectral density $S_{xx}(e^{j\omega})$ of the input sampled signal $x[n]$.

(ii) Plot the spectral density $S_{yy}(e^{j\omega})$ of the corresponding output signal $y[n]$.

(iii) Determine the numerical value of $\mathcal{E}\{y^2[n]\}$.

You MUST show all work to receive full credit.

(b) (12 pts) $x[n] = x_a(nT_s)$ where $T_s = \dfrac{2\pi}{40}$ and $r_{x_a x_a}(\tau) = \dfrac{2\pi}{40}\left\{\dfrac{\sin(10\tau)}{\pi\tau}\right\}$.

(c) (12 pts) $x[n] = x_a(nT_s)$ where $T_s = \dfrac{2\pi}{40}$ and $r_{x_a x_a}(\tau) = 2\dfrac{2\pi}{40}\left\{\dfrac{\sin(5\tau)}{\pi\tau}\right\}\cos(15\tau)$.

(d) (12 pts) $x[n] = x_a(nT_s)$ where $T_s = \dfrac{2\pi}{20}$ and $r_{x_a x_a}(\tau) = \dfrac{2\pi}{20}\left\{\dfrac{\sin(10\tau)}{\pi\tau}\right\}$.

(e) (12 pts) $x[n] = x_a(nT_s)$ where $T_s = \dfrac{3\pi}{20}$ and $r_{x_a x_a}(\tau) = \dfrac{3\pi}{20}\left\{\dfrac{\sin(10\tau)}{\pi\tau}\right\}$.

(f) (12 pts) $x[n] = x_a(nT_s)$ where $T_s = \dfrac{2\pi}{40}$ & $r_{x_a x_a}(\tau) = 1 + \cos(10\tau) + \cos(20\tau)$.

PROBLEM 2 ON NEXT PAGE.

Problem 2. [30 pts] Consider the discrete-time LTI system described by the following difference equation.

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3] \tag{2}$$

(a) (9 pts) Plot the magnitude $|H(e^{j\omega})|$ of the frequency response of this system equal to the DTFT $H(e^{j\omega})$ of the impulse response $h[n]$) as a function of frequency over $-\pi < \omega < \pi$. Show as much detail as possible.

(b) For $x[n] = x_a(nT_s)$ where $T_s = 3$ and $x_a(t) = u(t) - u(t-10)$, do the following:

(i) (7 pts) Plot the magnitude $|X(e^{j\omega})|$ of the DTFT of the sampled signal $x[n]$ which is input to the discrete-time system desribed by equation (2) above.

(ii) (7 pts) Plot the magnitude $|Y(e^{j\omega})|$ of the DTFT of the corresponding output signal $y[n]$.

(iii) (7 pts) Determine the numerical value of $\int_{-\pi}^{\pi} |Y(e^{j\omega})|^2 d\omega$. Show all work.