Q1 (15 points). Can the master method be applied to the recurrence $T(n) = 4T(n/2) + n^2 \lg n$? Why or why not? Give an asymptotic lower bound for this recurrence.

Q2 (30 points). A pipeline is to be built in Siberia and Prof. Khrochev is to decide on the optimal location of the stations along the path that has already been decided. More precisely, the pipeline connects points x_1 and x_n and goes through locations $x_2, x_3, \ldots, x_{n-1}$. There are stations at x_1 and x_n, but part of the problem is to decide whether to build a station at point x_i, $i = 2, 3, \ldots, n-1$. If a station is built in x_i there is an associated cost b_i, and if a pipeline section is built between stations x_i and x_j, with no other station in between, there is an associated cost c_{ij}. The total cost is the sum of the station costs plus the corresponding pipeline sections costs. The goal is to decide the optimal location of stations so that the total cost is minimized.

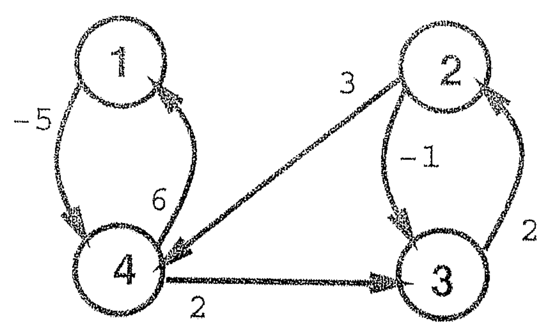
Describe a dynamic programming approach that Prof. Khrochev could use to make the decision (without any assumptions about the costs). Also describe an efficient pseudcode of your approach and argue that it is correct and analyze its running time.

Q 3 (25 points). Execute the FLOYD-WARSHALL algorithm on the weighted directed graph shown below after modifying the algorithm to include the computation of the $\Pi^{(k)}$ matrices according to the following equations.

$$\pi_{ij}^{(0)} = \begin{cases} NIL & \text{if } i = j \text{ or } w_{ij} = \infty, \\ i & \text{if } i \neq j \text{ and } w_{ij} < \infty. \end{cases}$$

$$\pi_{ij}^{(k)} = \begin{cases} \pi_{ij}^{(k-1)} & \text{if } d_{ij}^{(k-1)} \leq d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \\ \pi_{kj}^{(k-1)} & \text{if } d_{ij}^{(k-1)} > d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \end{cases}$$

Show all the $D^{(k)}$ matrices (for $k = 4$) as the algorithm proceeds through its iterations. In addition show all the $\Pi^{(k)}$ matrices.



Q4 (30 points). Suppose we are given a set of a jobs that must be run on a single machine. Each job i has a *release time* r_i when it is first available for processing; a *deadline* d_i by which it must be completed; and a *processing duration* t_i. We will assume that all of these parameters are natural numbers. In order to be completed, job i must be allocated a contiguous slot of t_i time units somewhere in the interval $[r_i , d_i]$. The machine can run only one job at a time. The question is: Can we schedule all jobs so that each completes by its deadline? We will call this an instance of *Scheduling with Release Times and Deadlines.*

Prove *Scheduling with Release Times and Deadlines* is *NP-complete.* (HINT: Subset-Sum is a known NP-Complete problem.)