Optimization

1. (20 pts) Employ the DFP method to construct a set of Q-conjugate directions using the function

$$\begin{aligned} f &= \frac{1}{2}\boldsymbol{x}^\top Q\boldsymbol{x} - \boldsymbol{x}^\top \boldsymbol{b} + c \\ &= \frac{1}{2}\boldsymbol{x}^\top \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \boldsymbol{x} - \boldsymbol{x}^\top \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 3, \end{aligned}$$

where $\boldsymbol{x}^{(0)}$ is arbitrary.

Warning: Only Q-conjugate directions generated using the DFP method will be accepted as a solution to this problem.

2. (20 pts)

$$\begin{aligned} \text{optimize} \quad & x_2^2 - x_3 - x_1 \\ \text{subject to} \quad & x_1^2 - x_2 = 0 \\ & x_3 - x_1^3 = 0. \end{aligned}$$

Use second order sufficient conditions to determine the nature of the optimizer.

3. (20 pts)

$$\begin{aligned} \text{minimize} \quad & J = \frac{1}{2}\sum_{k=0}^{2} u_k^2 \\ \text{subject to} \quad & \boldsymbol{x}_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}_k + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_k, \\ & \boldsymbol{x}_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{x}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \end{aligned}$$

4. (20 pts)

$$\begin{aligned} \text{optimize} \quad & x_1^2 + 6x_1x_2 - 4x_1 - 2x_2 + 3 \\ \text{subject to} \quad & x_1^2 + 2x_2 \le 1 \\ & 2x_1 - 2x_2 \le 1. \end{aligned}$$

Try $\mu_1 = 0$, $\mu_2 \neq 0$. Use the second-order conditions to detrmine the nature of the solution.

5. (20 pts)

(i) **(10 pts)** Determine if the function, $f(x) = -8x^2$, is convex or concave?

(ii) **(10 pts)** Determine if the function, $f(x_1, x_2) = 2x_1^3 - 3x_2^2$, is convex or concave? Is there a subset of $\mathbb{R}^2$ over which this function is convex or concave?