

AC-1  
August 2007

## Linear Systems

1. (10 pts) Find the transfer function  $X(s)/U(s)$  of the block diagram, shown in Figure 1.

$X(s)/U(s) =$

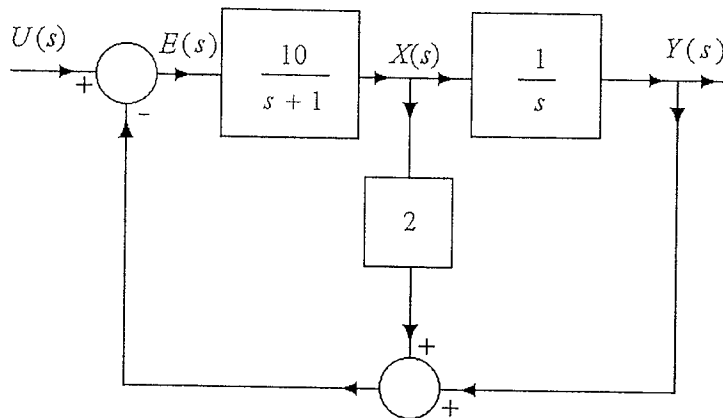


Figure 1: Block diagram for Problem 1.

2. (15 pts) For the closed-loop system, shown in Figure 2, find the settling time, using the 2% criterion (in secs).

$t_s =$

3. (15 pts) A negative unity feedback closed-loop system has the open-loop transfer function

$$G(s) = \frac{3}{s(s+1)^2}$$

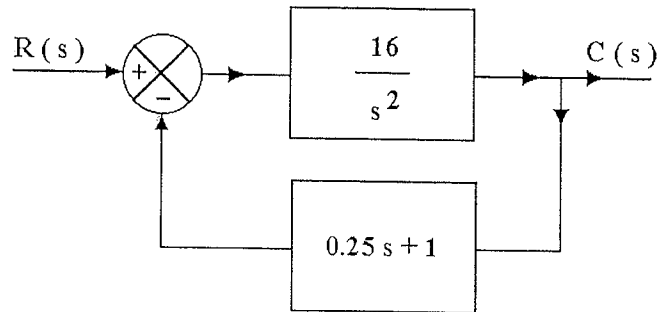


Figure 2: Closed-loop system for Problem 2.

Find the gain margin,  $K_g$ , of the closed-loop system in numbers.

$K_g =$

4. (15 pts) We wish the dominant complex conjugate poles of the closed-loop system to be located at  $-1 \pm j2$ . The deficiency angle of the uncompensated closed-loop system at the upper dominant pole is  $45^\circ$ . We use a PD compensator in the forward path. The transfer function of the compensator is

$$G_c(s) = K_c(1 + T_d s).$$

Find  $T_d$  that achieves this design goal.

$T_d =$

5. (15 pts) We wish the dominant complex conjugate poles of the closed-loop system to be located at  $-4 \pm j2\sqrt{3}$ . The argument of the open loop-transfer function evaluated

at the upper dominant pole is  $150^\circ$ . We use a lead compensator in forward path. The transfer function of the compensator is

$$G_c(s) = K_c \frac{s - z_c}{s - p_c}.$$

We set  $z_c = -4$ . Find  $p_c$ .

$p_c =$

6. (15 pts) The velocity error coefficient of an uncompensated unity feedback closed-loop system is  $K_v = 2.5$ . We wish to improve the steady-state performance of the closed-loop system by placing a lag compensator in the forward path. The transfer function of the compensator is

$$G_c(s) = \hat{K}_c \beta \frac{0.1s + 1}{0.1\beta s + 1}.$$

To keep the dominant closed-loop poles in the desired location, we use the gain  $\hat{K}_c = 2$ . The desired velocity error coefficient of the compensated closed-loop system is  $\hat{K}_v = 40$ . Find  $\beta$  that achieves this design goal.

$\beta =$

7. (15 pts) In Figure 3, on the next page, Bode diagrams are shown of the uncompensated open-loop system with the gain adjusted for the desired error coefficient. It is required that the phase margin of the compensated closed-loop system, using a lag compensator, be 55 degrees. Add 5 deg to compensate for the phase lag of the compensator. Select the corner frequency corresponding to the zero of the compensator one decade below the new gain crossover frequency. Find the location of the zero of the lag compensator.

$z_c =$

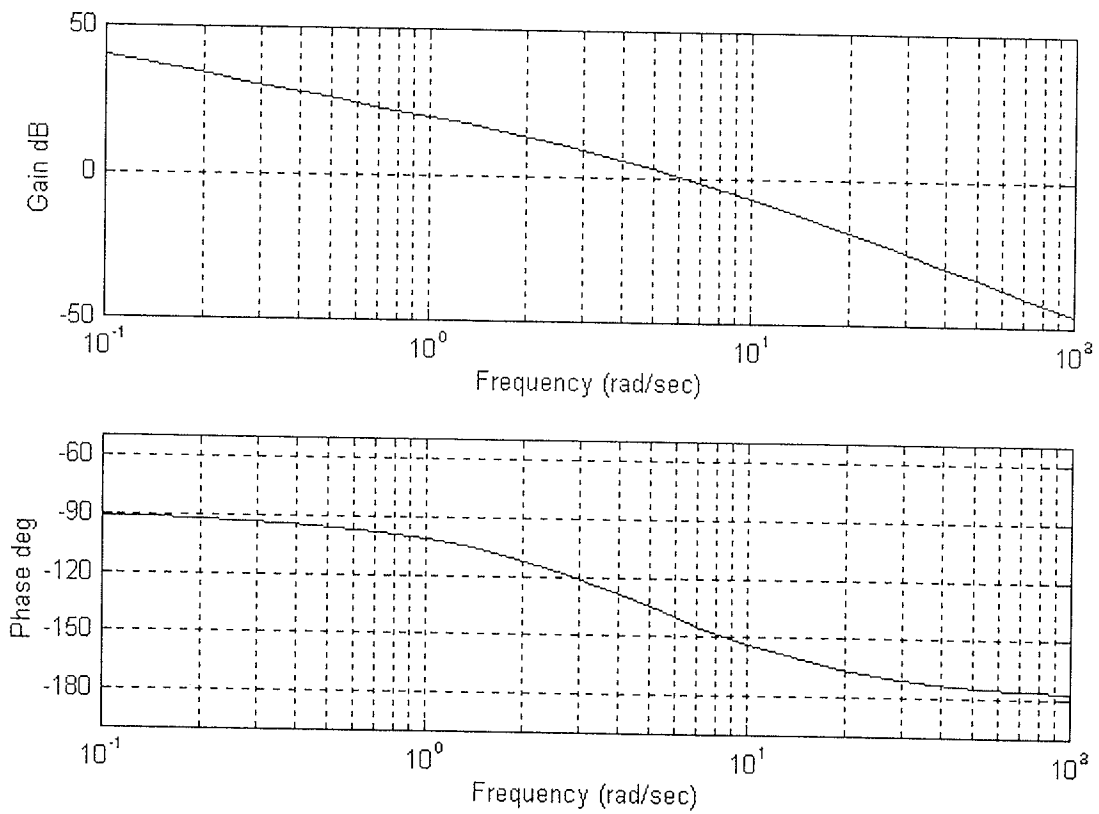


Figure 3: Bode diagrams for Problem 7.