PE-2
August 2017 QE

Problem 1. [50 points] Consider a symmetrical three-phase 6-pole wound-rotor induction machine, as shown in Fig. 1. Torque is assumed positive in the counterclockwise direction. Recall that

$$T_e = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right)$$
$$= \frac{3}{2}\frac{P}{2}\left(\lambda'_{qr} i'_{dr} - \lambda'_{dr} i'_{qr}\right)$$

and that the transformation of the variables of a circuit to the arbitrary reference frame, where the circuit angle is θ_c and the reference frame angle is θ, involves multiplication by

$$\mathbb{K}(\beta) = \frac{2}{3}\begin{bmatrix} \cos\beta & \cos(\beta - 2\pi/3) & \cos(\beta + 2\pi/3) \\ \sin\beta & \sin(\beta - 2\pi/3) & \sin(\beta + 2\pi/3) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

where $\beta = \theta - \theta_c$. For the following questions, the trigonometric identities found in the last page of this question may be useful.

(a) The winding currents are $\mathbf{i}_{abcs} = \begin{bmatrix} 0 & I_1 & -I_1 \end{bmatrix}$ and $\mathbf{i}'_{abcr} = \begin{bmatrix} 0 & -I_2 & I_2 \end{bmatrix}$, where I_1 and I_2 are positive constants. Sketch the electromagnetic torque vs. θ_{rm} over 360 mechanical degrees.

(b) The winding currents are $\mathbf{i}_{abcs} = \begin{bmatrix} I_1 & I_1 & I_1 \end{bmatrix} \cdot \cos 10t$ and $\mathbf{i}'_{abcr} = \begin{bmatrix} 0 & -I_2 & I_2 \end{bmatrix}$, where I_1 and I_2 are positive constants. The electrical rotor angle is $\theta_r = 90°$ (constant). Calculate the electromagnetic torque as a function of time.

(c) Given the current waveforms and rotor angle of part (b), write expressions for $v^s_{qs}(t)$ and $v'^s_{qr}(t)$ in a stationary reference frame with $\theta = 0$. The expressions may contain the machine parameters, but they must be as simple as possible given the information provided.

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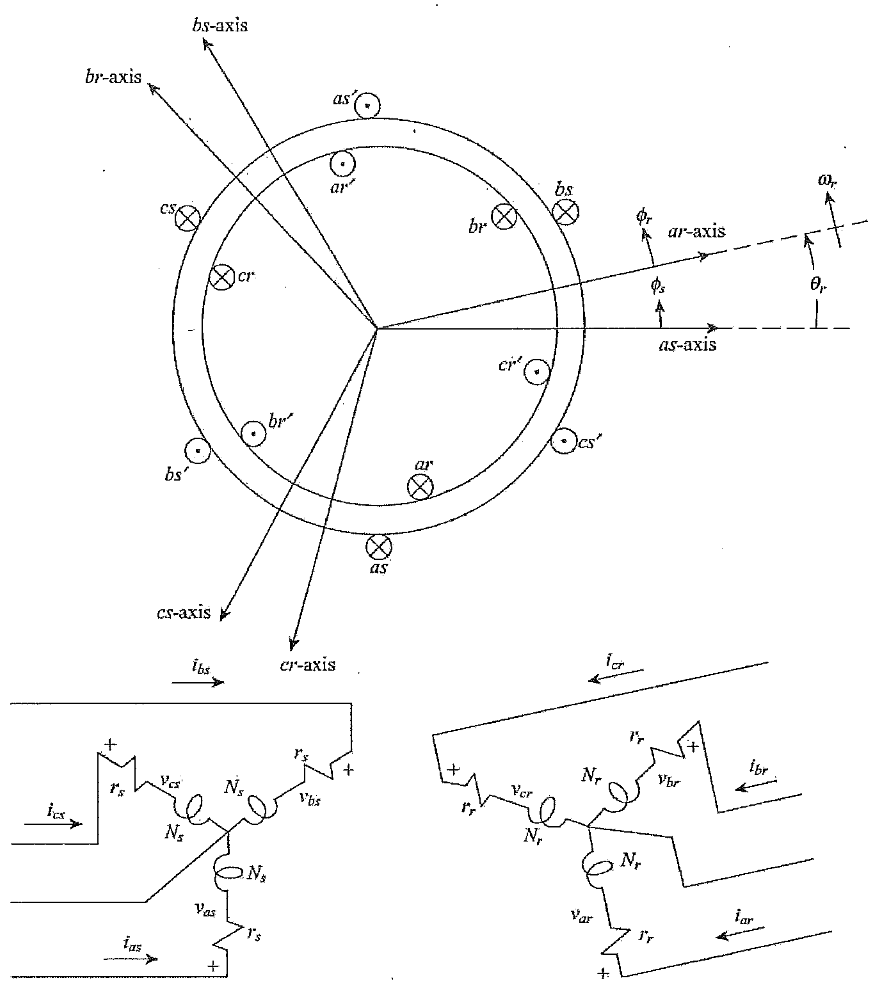

Figure 1: Elementary induction machine cross-section and winding interconnection diagram.

Problem 2. [50 points] Consider a three-phase Y-connected salient-pole wound-rotor synchronous machine. Steady-state operating conditions are to be assumed. Use generator convention for currents. Throughout this problem, the following operating conditions apply:

- The machine is connected directly to an "infinite bus", i.e., a constant-frequency and constant-voltage three-phase balanced source.
- The excitation is kept constant at the value which would result in 1.0 power factor at rated power when acting as a generator.

Other data are provided in the Table below.

parameter	value
rated power	1.5 MVA
stator resistance	$r_s = 0.0\ \Omega$
d-axis reactance	$X_d = 2.0\ \Omega$
q-axis reactance	$X_q = 1.0\ \Omega$
terminal voltage	$V_t = 2000$ V, line-to-neutral, peak
source frequency	$\omega_e = \omega_b$ (i.e., nominal)

For the following questions, you may use the plots of $\sin x$, $\arctan x$, and $\sqrt{x}$ on the next pages (Figs. 3–5) to help you with the calculations. Depending on the approach that you take, you may not need all of these. Approximations to one decimal digit are acceptable. You may leave $\sqrt{2}$ or $\sqrt{3}$ factors in the expressions.

(a) For the operating conditions described above, calculate the a-phase current phasor, $\tilde{I}_{as}$.

(b) For the operating conditions described above, calculate the internal voltage phasor $\tilde{E}_a$ (preferably in polar form), defined by the following textbook formulas for steady-state operation (using generator convention):

$$\tilde{V}_{as} = -\left(r_s + j\frac{\omega_e}{\omega_b}X_q\right)\tilde{I}_{as} + \tilde{E}_a$$

$$\tilde{E}_a = \frac{1}{\sqrt{2}}\left[\frac{\omega_e}{\omega_b}(X_q - X_d)I_{ds}^r + \frac{\omega_e}{\omega_b}X_{md}I_{fd}'^r\right]e^{j\delta}$$

$$\sqrt{2}\tilde{F}_{as}e^{-j\delta} = F_{qs}^r - jF_{ds}^r$$

(c) For the operating conditions described above, calculate the d-axis current, I_{ds}^r.

(d) For the operating conditions described above, calculate the excitation voltage, $E_f = X_{md}I_{fd}'^r$.

(e) It is known that the electric power output (positive for generator action) can be expressed as

$$P_s = P_{f,\max}\sin\delta + P_{r,\max}\sin 2\delta \qquad (\dagger)$$

where

$$P_{f,\max} = \frac{3}{2}\frac{E_f V_t}{X_d} \quad \text{and} \quad P_{r,\max} = \frac{3}{2}V_t^2\frac{X_d - X_q}{2X_d X_q}.$$

Equation (†) yields a normalized power equation

$$\frac{P_s}{P_{f,\max}} = \sin\delta + \frac{P_{r,\max}}{P_{f,\max}} \sin 2\delta\,.$$

This is plotted in Fig. 2 as a function of δ for various values of $P_{r,\max}/P_{f,\max}$. Based on Fig. 2, compute the maximum power that this machine can deliver acting as a motor, assuming that the shaft load is increased gradually so that transient swings are negligible and steady-state conditions apply. Friction and other losses can be ignored.

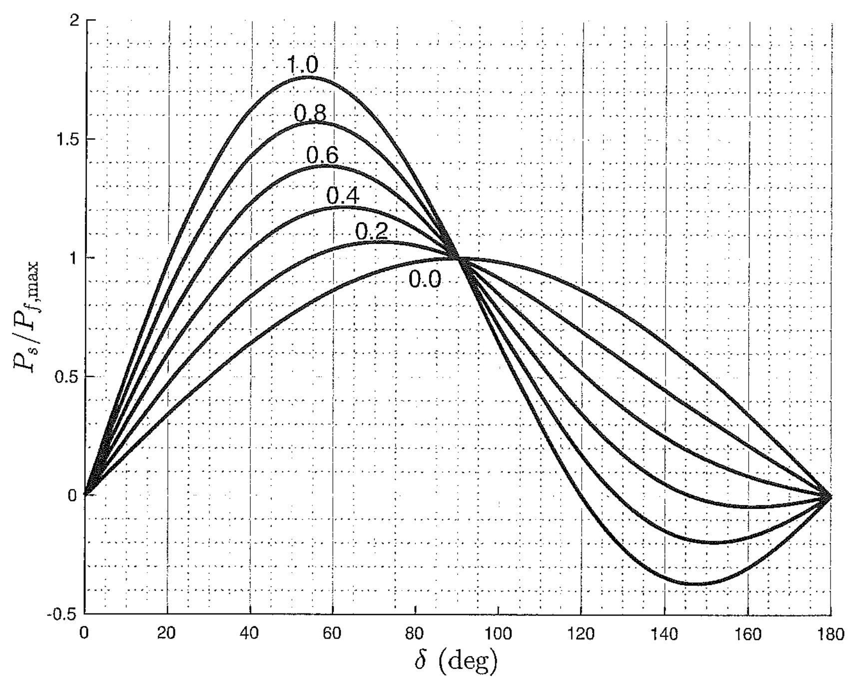

Figure 2: Plots of normalized power equation as a function of δ with parameter $P_{r,\max}/P_{f,\max} \in \{0.0, 0.2, \ldots, 1.0\}$.

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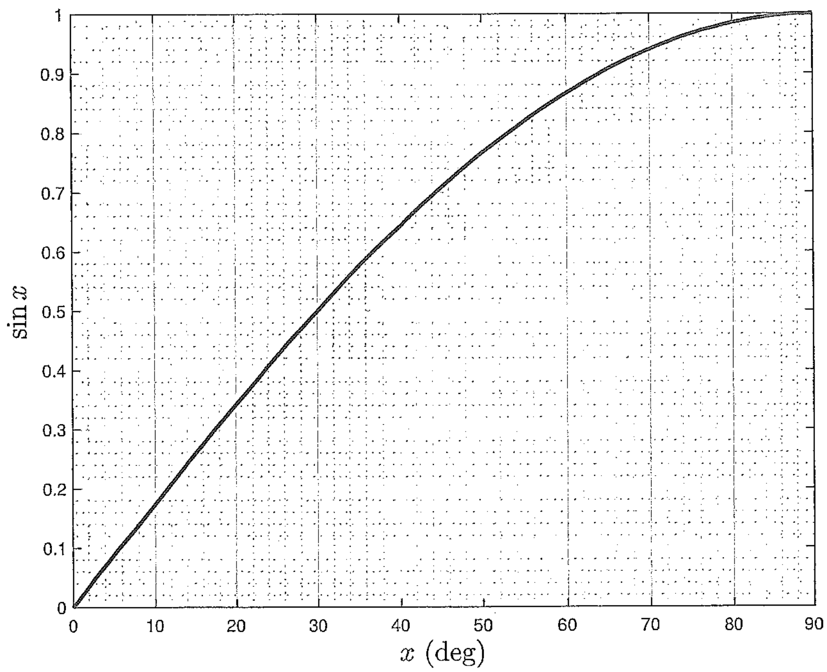

Figure 3: Plot of $\sin x$ for $x \in [0, \pi/2]$.

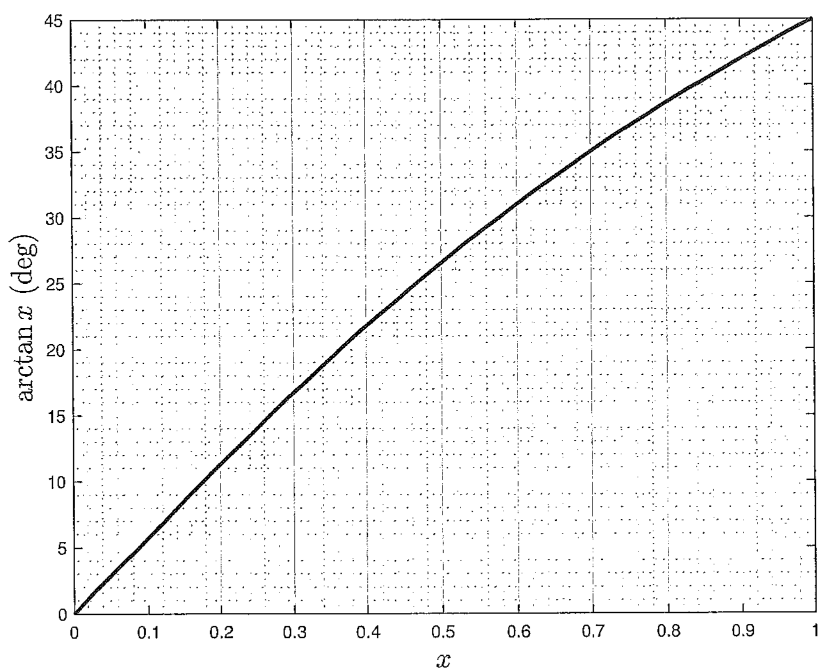

Figure 4: Plot of $\arctan x$ for $x \in [0, 1]$.

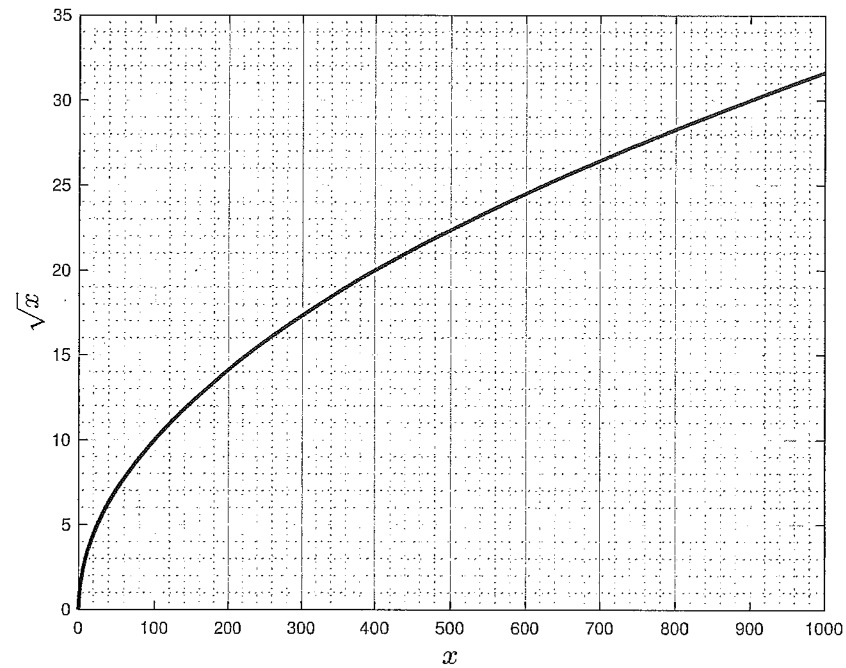

Figure 5: Plot of $\sqrt{x}$ for $x \in [0, 1000]$.

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Potentially useful trigonometric identities:

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\sin(\tfrac{\pi}{2} - x) = \cos x$$
$$\sin(x + \pi) = -\sin x$$
$$\cos(x + \pi) = -\cos x$$
$$\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$$
$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$
$$\cos\theta + \cos\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\cos\left(\frac{\theta-\phi}{2}\right)$$
$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$
$$\sin\theta - \sin\phi = 2\cos\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$
$$2\sin a\cos b = \sin(a+b) + \sin(a-b)$$
$$2\cos a\cos b = \cos(a+b) + \cos(a-b)$$
$$-2\sin a\sin b = \cos(a+b) - \cos(a-b)$$
$$2\cos^2 x = \cos 2x + 1$$
$$2\sin^2 x = 1 - \cos 2x$$
$$\cos x + \cos\left(x - 2\pi/3\right) + \cos\left(x + 2\pi/3\right) = 0$$
$$\sin x + \sin\left(x - 2\pi/3\right) + \sin\left(x + 2\pi/3\right) = 0$$