You must show work for credit

Problem 1. 33 pts. The torque of a magnetically linear single-phase electromechanical device may be expressed

\[ T_r = 8\cos(4\theta_{rm})i^2 \]

When the rotor position is zero, the inductance is 5 H. What is the flux linkage equation for the device?

Problem 2. 33 pts. The \( \alpha \)-phase conductor density of a machine is given by

\[ n_{a\alpha} = -6\cos(4\phi_{sm}) \]

where \( \phi_{sm} \) is position measured relative to the stator. The radial flux density at the stator inner radius is given by

\[ B(\phi_{rm}) = \frac{2}{3}\cos(4\phi_{rm}) \]

where \( \phi_{rm} \) is position measured relative to the rotor. The machine is 0.1 m long. With the stated flux density, and neglecting winding resistance and leakage inductance, it is desired that the \( \alpha \)-phase voltage is a 100 V zero-to-peak sinewave when the mechanical rotor speed is \( 1000/\pi \) rad/s. What should be the radius (stator inner radius) of the machine?

Handy Fact: \( 2\sin A\cos B = \sin(A + B) + \sin(A - B) \)

Problem 3. 34 pts. Transform the following flux linkage equation to the rotor reference frame:

\[ \lambda_{a\alpha\beta\gamma} = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -1 \\ -1 & -1 & 4 \end{bmatrix} i_{a\beta\gamma} + 2 \begin{bmatrix} \cos(\theta_r) \\ \cos(\theta_r - 2\pi/3) \\ \cos(\theta_r + 2\pi/3) \end{bmatrix} \]

Handy facts:

\[ \cos(x) + \cos(x - 2\pi/3) + \cos(x + 2\pi/3) = 0 \]
\[ \sin(x) + \sin(x - 2\pi/3) + \sin(x + 2\pi/3) = 0 \]
\[ \cos(x)\cos(y) + \cos(x - 2\pi/3)\cos(y - 2\pi/3) + \cos(x + 2\pi/3)\cos(y + 2\pi/3) = \frac{3}{2}\cos(x - y) \]
\[ \sin(x)\sin(y) + \sin(x - 2\pi/3)\sin(y - 2\pi/3) + \sin(x + 2\pi/3)\sin(y + 2\pi/3) = \frac{3}{2}\cos(x - y) \]
\[ \sin(x)\cos(y) + \sin(x - 2\pi/3)\cos(y - 2\pi/3) + \sin(x + 2\pi/3)\cos(y + 2\pi/3) = \frac{3}{2}\sin(x - y) \]