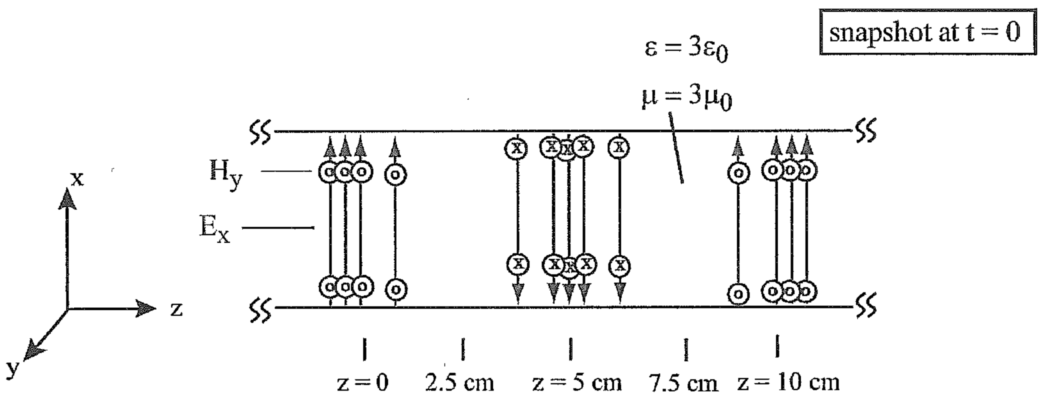
FO-3
August 2017 QE

1. (30 points)

A time-varying electromagnetic wave propagates in a lossless parallel plate transmission line consisting of two perfectly conducting metal plates separated by a lossless spacer material. The plates are much wider in the transverse direction (out of the paper, not shown) compared to their vertical separation. The spacer material between the plates is linear with electric permittivity $\varepsilon = 3\varepsilon_0$ and magnetic permeability $\mu = 3\mu_0$. The figure shows a snapshot of the electric field and magnetic field lines at a fixed instant of time t = 0. The **E** field is oriented along x, and the **H** field is oriented along y. Note the spatial scale along z that is included in the figure.



⊙ : out of paper

⊗ : into paper

8 points a) Is the electromagnetic wave propagating in the +z or –z direction? How do you know? (No credit without a clear explanation).

12 points b) In your solution book, make a neat copy of the figure including the **E** and **H** lines. Then clearly add to the sketch the charges and currents that accompany the **E** and **H** fields in the transmission line. Be sure to indicate all the locations of charge and current. Also indicate the signs of the charges and the directions of the currents. Give a brief justification for your answer.

10 points c) What is the frequency of the electromagnetic field in Hertz (NOT the angular frequency in rad/sec)? Please provide an answer both in terms of a formula as well as an approximate numerical value. (The algebra should be simple even without a calculator.)

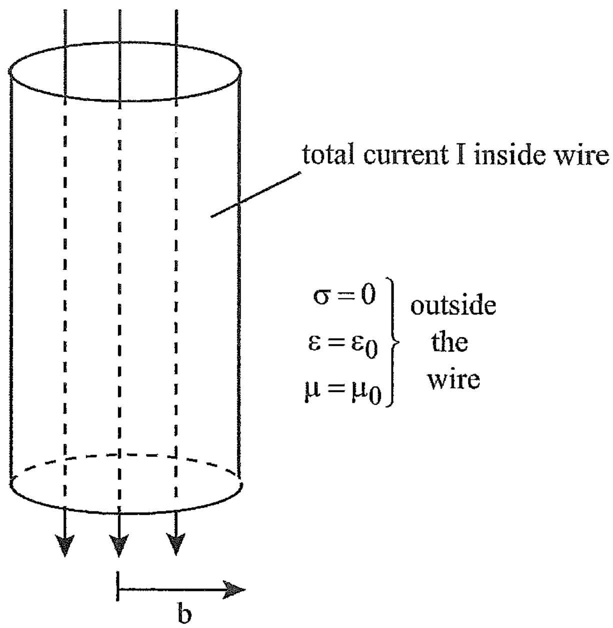
2. (35 points)

A very long, straight conducting wire (with radius b, conductivity σ, electric permittivity ε, and magnetic permeability μ) carries a direct (time independent) current I. The current is distributed uniformly inside the wire. The electrical charge density is zero everywhere.

Directly compute each of the terms in Poynting's theorem applied to a cylinder of radius b and height h and confirm that Poynting's theorem holds.

Poynting's theorem in a linear, isotropic medium may be expressed as:

$$\oint_S \mathbf{E}\times\mathbf{H}\cdot d\mathbf{S} = -\int dV\left\{\frac{\partial}{\partial t}\left(\frac{1}{2}\mu\mathbf{H}\cdot\mathbf{H}+\frac{1}{2}\varepsilon\mathbf{E}\cdot\mathbf{E}\right)+\mathbf{E}\cdot\mathbf{J}\right\}$$

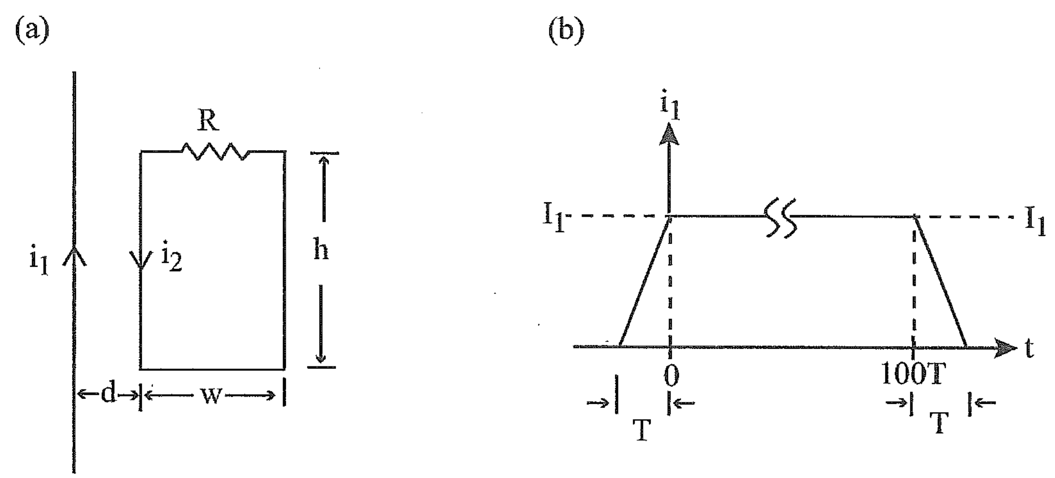


Write in Exam Book Only

3. (35 points)

A rectangular loop of width w and height h is situated a distance d away from a very long wire carrying a current $i_1(t)$ as shown in the figure, part (a). The loop is made up of a resistor R connected at both ends to thin, perfectly conducting wire. Assume $i_1(t)$ to be a current pulse of amplitude I_1, with linear rising and falling edges, each of duration T, and a flat top of duration 100T – see figure, part (b).

Assume that the self-inductance of the loop L is very small, such that the L/R time constant of the loop is much less than t, i.e., $\frac{L}{R} << T$, and can be ignored.



15 points a) Find the induced current $i_2(t)$ in the rectangular loop. Specify $i_2(t)$ for all times t.

10 points. b) Find the energy dissipated in the resistor.

10 points. c) Explain why the assumption L/R << T simplifies the problem. What would change in your analysis if L/R is not small compared to T? A brief but clear response in words is sufficient; no mathematics are required.