Problem 1. Suppose that one of the baseband signals, \( s_0(t) = T \rho_T(t) \) or \( s_1(t) = (T - t) \rho_T(t) \), is received in the presence of additive white Gaussian noise (AWGN) \( n(t) \) with two-sided power spectral density \( N_0/2 \). (The function \( \rho_T(t) \) is defined by \( \rho_T(t) = 1 \) for \( 0 \leq t < T \), and \( \rho_T(t) = 0 \), elsewhere.) The receiver is shown below.

![Diagram of receiver](image)

Figure 1: Receiver Used for Problem (1) with Sampling Modified in Part (c).

The sum of the signal \( s_i(t) \), where \( i = 0 \) or \( i = 1 \), and noise \( n(t) \) passes through a filter with impulse response \( h(t) \), and the output of the filter is sampled at time \( T_0 \) to produce a decision statistic \( \hat{r}(T_0) \). If \( \hat{r}(T_0) \geq \gamma \), the receiver decides that \( s_0(t) \) was sent, and if \( \hat{r}(T_0) < \gamma \), the receiver decides that \( s_1(t) \) was sent. Let \( P_{e,0} \) denote the probability of error given that \( s_0(t) \) is sent and \( P_{e,1} \) denote the probability of error given that \( s_1(t) \) is sent. (When necessary, express answers in terms of \( \Phi(x) \), the cumulative distribution function of a zero-mean, unit-variance Gaussian random variable.)

(a) (25 pts.) Find the impulse response \( h(t) \) of the filter, the threshold \( \gamma \), and the sample time \( T_0 \) that minimize the maximum of \( P_{e,0} \) and \( P_{e,1} \).

(b) (20 pts.) Now suppose that \( h(t) = \rho_T(t) \), \( T_0 = T \), and \( \gamma = 0 \). Find both \( P_{e,0} \) and \( P_{e,1} \).

(c) (25 pts.) Now, again suppose that \( h(t) = \rho_T(t) \) and \( \gamma = 0 \). However, a modification of the receiver shown above is made so that the output of the filter is sampled twice, once at \( t = T/2 \) and once at \( t = T \). The samples are then added to form the decision statistic \( \hat{r}(T/2) + \hat{r}(T) \) and compared with the threshold \( \gamma = 0 \). If \( \hat{r}(T/2) + \hat{r}(T) \geq 0 \), the receiver decides that \( s_0(t) \) was sent, and if \( \hat{r}(T/2) + \hat{r}(T) < 0 \), the receiver decides that \( s_1(t) \) was sent. Find \( P_{e,0} \) with this new detection scheme.
**Problem 2.** Suppose that only additive white Gaussian noise (AWGN) with two-sided power spectral density $N_0/2 = 5 \times 10^{-6} \text{ W/Hz}$ is applied to the input of an ideal differentiator as depicted in the figure shown below:

![Diagram of Ideal Differentiator](image)

Figure 2: Diagram of Ideal Differentiator for Problem (2).

(a) (10 pts.) Write down an expression for the transfer function $H(f)$ of the ideal differentiator.

(b) (20 pts.) Find the power of the output of the ideal differentiator that is present in the frequency band $100 < |f| < 200$, where $f$ is given in Hertz. (For partial credit, at least express this answer in terms of the transfer function $H(f)$.)