Q1 (20 points). Let $b > 1$ be a constant. Prove or dis-prove that $O\left(t(n)\right) \times O\left(b^{t(n)}\right) = 2^{o(t(n))}$.

Q2 (28 points). You bring an $\ell$-foot log of wood to your favorite sawmill. You want it cut in $k$ specific places: $\ell_1$, $\ell_2$, ..., $\ell_k$ feet from the left end. The sawmill charges $x$ to cut an $x$-foot log any place you want.

(a) (18 points). Devise a recurrence that characterizes the optimal solution that determines the order in which the sawmill should cut your log in order to minimize the cost. The running time of your algorithm should be polynomial in $k$. Analyze the running time.

(b) (10 points). Consider a greedy algorithm that cuts the wood such that the maximum length of the resulting two pieces is always as small as possible. Prove this algorithm does not achieve the minimal cost. (Hint: use a counter example).

Q3 (20 points) Prove a graph has a unique Minimum Spanning Tree if all the edges in the graph have distinct weights.

Q4 (25 points) A triangle in an undirected graph is a 3-clique. Show that $\text{TRIANGLE} \in \text{P}$, where $\text{TRIANGLE} = \{ \langle G \rangle | G \text{ contains a triangle} \}$.

Q5 (7 points) Circle the right Yes/No choice. 3-SAT is

(a) P  
(b) NP  
(c) CoNP  
(d) NP-Hard  
(e) CoNP-Hard  
(f) NP-Complete  
(g) CoNP-Complete