1. (40 pts) Compare the energy of the following configurations. The permittivity $\varepsilon$ is $\varepsilon_0$ everywhere.
   
   a. In (A), a point charge $Q$ is suspended a distance $d$ above a grounded plane. In (B), two point charges, $+Q$ and $-Q$, are separated by a distance $2d$. How does the energy $W_A$ of configuration (A) compare to that of configuration (B), $W_B$? Specifically, find $W_A/W_B$. Explain your reasoning.

   ![Diagram of configurations A and B]

   b. In these configurations, the two spheres are of radius $a$, and each contains a total charge of $Q$. In (A), the volume charge density $\rho_v$ is uniform in the region $R < a$, and zero everywhere else. In (B), the only charge is a surface charge, of density $\rho_s$, uniformly distributed on the surface of the sphere. (i) Compare and contrast the electric field in the regions $R < a$ and $R > a$ for these two charge configurations. [Note: No equations are necessary for your response to this part of the question.] (ii) Explain qualitatively the difference between the energy $W_A$ of configuration (A) and the energy $W_B$ of configuration (B). (iii) Find $W_A/W_B$.

2. (30 pts) A parallel plate capacitor of spacing $d$, is charged to a potential $V_0$. The dimension of the conductors is $a \times b$, where $b$ is the dimension into the page. A dielectric slab, of permittivity $\varepsilon = 2\varepsilon_0$, is partially inserted into the region between the plates, as shown. Neglecting edge effects, approximate the force $F$ felt by the dielectric. Is the force pulling the dielectric into the space (as shown), or expelling the dielectric (opposite the direction shown). Explain your answer. [Hint: Recall that the mechanical work done in moving an object against a force $F$ is $-\int F \cdot d\ell$. Except for consideration of signs, the inverse of this relation can be used to find the force $F$.]

![Diagram of capacitor with dielectric]
3. (30 pts) Consider a charged region of infinite length in the $x$ and $y$ dimensions. The displacement field $\mathbf{D}$ has only a $D_z$ component, which is $D_z(z) = A + Bz$ for $0 < z < d$, $D_z(z) = C$ for $z > d$, and $D_z(z) = -D_z(z)$. $A$, $B$, and $C$ are known constants. See the plot to the right.

a. Determine the volume charge density $\rho_V$ for the four regions $0 < z < d$, $z > d$, $z < -d$, and $-d < z < 0$.

b. Determine the surface charge density $\rho_s$ at $z = d$.

c. Determine the total charge $Q$ contained within a volume $V$ which has surface area $\Delta S$ on the faces normal to the $z$-axis, and extends over $-L < z < L$, where $L > d$, in the $z$-direction.
Maxwell's Equations:

\[ \nabla \cdot \mathbf{D} = \rho_v \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]

Poynting's Theorem:

\[ \nabla \cdot (E \times H) = -\frac{\partial}{\partial t} \left( \frac{1}{2} \mu \mathbf{H} \cdot \mathbf{H} \right) - \frac{\partial}{\partial t} \left( \frac{1}{2} \varepsilon \mathbf{E} \cdot \mathbf{E} \right) - \mathbf{J} \cdot \mathbf{E} \]

Potentially useful vector algebra

\[ \nabla \mathbf{A} = \hat{x} \left( \frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \hat{y} \left( \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \hat{z} \left( \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial y} \right) \]
\[ \nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \]
\[ \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \]

Potentially useful integral identities

\[ \int \frac{dx}{x} = \ln x \]
\[ \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( x + \sqrt{x^2 + a^2} \right) \]
\[ \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \]
\[ \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} \]
\[ \int \sin^2 x \, dx = \frac{x}{2} - \frac{1}{4} \sin 2x \]
\[ \int \sin^3 x \, dx = \frac{\cos^3 x}{3} - \cos x \]
\[ \int \sinh^2 x \, dx = \frac{1}{2} [-x - \sinh x \cosh x] \]
\[ \int \frac{xdx}{\sqrt{x^2 + a^2}} = \sqrt{x^2 + a^2} \]
\[ \int \frac{xdx}{x^2 + a^2} = \frac{1}{2} \ln \left( x^2 + a^2 \right) \]
\[ \int \left( x^2 + a^2 \right)^{-1/2} \, dx = \frac{1}{\sqrt{x^2 + a^2}} \]
\[ \int \cos^2 x \, dx = \frac{x}{2} + \frac{1}{4} \sin 2x \]
\[ \int \cos^3 x \, dx = -\frac{\sin^3 x}{3} + \sin x \]
\[ \int \cosh^2 x \, dx = \frac{1}{2} [x + \sinh x \cosh x] \]