Problem 1. (50pt)
Consider the emissive display device which is accurately modeled by the equation

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = \begin{bmatrix}
a & b & c \\
d & e & f \\
g & h & i
\end{bmatrix} \begin{bmatrix}
R^\alpha \\
G^\alpha \\
B^\alpha
\end{bmatrix}
\]

where \( R, G, \) and \( B \) are the red, green, and blue inputs in the range 0 to 255 that are used to modulate physically realizable color primaries.

a) (10pt) What is the gamma of the device?

b) (10pt) What are the chromaticity components \((x_r, y_r), (x_g, y_g),\) and \((x_b, y_b)\) of the device’s three primaries.

c) (10pt) What are the chromaticity components \((x_w, y_w)\) of the device’s white point.

d) (10pt) Sketch a chromaticity diagram and plot and label the following on it:

1. \((x, y) = (1, 0)\)
2. \((x, y) = (0, 1)\)
3. \((x, y) = (0, 0)\)
4. \((R, G, B) = (255, 0, 0)\)
5. \((R, G, B) = (0, 255, 0)\)
6. \((R, G, B) = (0, 0, 255)\)

e) (10pt) Imagine that the values of \((R, G, B)\) are quantized to 8 bits, and that you view a smooth gradient from black to white on this device. What artifact are you likely to see, and where in the gradient will you see it?
Problem 2. (50pt)
Consider an X-ray imaging system shown in the figure below.

![Diagram](image)

Photons are emitted from an X-ray source and columnated by a pin hole in a lead shield. The columnated X-rays then pass in a straight line through an object of length $T$ with density $u(x)$ where $x$ is the depth into the object. The number of photons in the beam at depth $x$ is denoted by the random variable $Y_x$ with Poisson density given by

$$P\{Y_x = k\} = \frac{e^{-\lambda_x} \lambda_x^k}{k!}.$$

where $x$ is measured in units of $cm$ and $\mu(x)$ is measured in units of $cm^{-1}$.

a) (10pt) Calculate the mean of $Y_x$, i.e., $E[Y_x]$.

b) (10pt) Calculate the variance of $Y_x$, i.e., $E[(Y_x - E[Y_x])^2]$.

c) (10pt) Write a differential equation which describes the behavior of $\lambda_x$ as a function of $x$.

d) (10pt) Solve the differential equation to form an expression for $\lambda_x$ in terms of $u(x)$ and $\lambda_0$.

e) (10pt) Calculate an expression for the integral of the density, $\int_0^T u(x) dx$, in terms of $\lambda_0$ and $\lambda_T$. 