1. Consider the single-phase full-bridge rectifier

Assume the dc current $I_{dc}$ is constant.

(a) Derive an expression for commutation angle $\gamma$ in terms of $E_p$, $\omega_c$, $I_{dc}$, and $I_c$.

(b) Derive an expression for the average dc voltage in terms of $E_p$ and $\gamma$.

2. Consider the four-quadrant dc motor drive system

Assume switches and diodes are ideal.

(a) If $\omega_r = 100$ rad/s, establish average $T_e$.

(b) Suppose the minimum value of steady-state $i_a(t) = 4$ A, the maximum value of steady-state $i_a(t) = 6$ A, and $T \ll \tau$. Sketch steady-state $i_{s1}(t)$, $i_{D1}(t)$, and $i_S(t)$ for $0 < t < 100 \mu s$ and approximate their average values.
3. Consider the three-phase drive system

(a) Assume \( S_{2a} \) is closed when \( S_{1a} \) is open and vice versa. A sine-triangle modulation strategy is used wherein the modulating (triangle) frequency is much larger than fundamental frequency \( \omega_c \). The duty cycle for each phase is

\[
\begin{align*}
    d_a(t) &= d \cos \omega_c t \\
    d_b(t) &= d \cos (\omega_c t - \frac{2\pi}{3}) \\
    d_c(t) &= d \cos (\omega_c t + \frac{2\pi}{3}).
\end{align*}
\]

Express the “fast” or “moving” averages \( \hat{\varphi}_{ag}(t) \), \( \hat{\varphi}_{bg}(t) \), and \( \hat{\varphi}_{cg}(t) \) in terms of \( d \), \( V_{dc} \), and \( \omega_c \). Then, derive an expression for \( \hat{v}_{anc}(t) \). Assume the zero-sequence component of \( v_{an}, v_{bn}, \) and \( v_{cn} \) is zero.

(b) The duty cycle for each phase is

\[
\begin{align*}
    d_a &= d \cos \theta_c - d_3 \cos 3\theta_c \\
    d_b &= d \cos (\theta_c - \frac{2\pi}{3}) - d_3 \cos 3\theta_c \\
    d_c &= d \cos (\theta_c + \frac{2\pi}{3}) - d_3 \cos 3\theta_c.
\end{align*}
\]

Express \( \hat{v}_{anc}(t) \) in terms of \( d \), \( d_3 \), and \( V_{dc} \). What is the rationale for including third-harmonic injection \( d_3 \)? If \( d_3 = d/6 \), what is the maximum value of \( d \) for which your expression for \( \hat{v}_{anc}(t) \) is valid?