1. For 3D crystalline silicon, near the bottom of the conduction band and close to the top of the valence band, we can consider parabolic energy dispersions, as shown in the E-k dispersion below. Assume the hole effective mass is $m_h$, and the electron effective mass is $m_e$.

![Parabolic Energy Dispersion](image)

a) Find the density of states expressions for both electrons and holes. (20pts)

b) Make appropriate approximations to the Fermi Distribution, $f = \frac{1}{1 + e^{(E - E_F)/k_B T}}$, considering $E - E_F \gg k_B T$, find an expression for the following function:

$$g(T) = np$$

where $n$ and $p$ are the electron and hole concentrations and $T$ is the temperature. (You may need to know $\int_0^\infty x^{1/2} e^{-x} \, dx = \left(\frac{\pi}{4}\right)^{1/2}$) (20pts)

c) What concentration $N_D$ of As donors must be used to make the conductivity of Si 100 times greater than the intrinsic conductivity at room temperature? Assume donors are fully ionized. It is known that the carrier concentration of intrinsic Si is $n_i \approx 10^{10}/\text{cm}^3$ at room temperature. In your conductivity calculations, neglect acceptor impurities and consider the electron and hole mobility to be the same in intrinsic Si. (10pts)
2. The work function $\phi_s$ of a semiconductor is the difference in energy between an electron at rest in vacuum and the Fermi energy in the semiconductor. If a metal with work function $\phi_m$ is used to make contact with the semiconductor, you can use band diagrams of the materials at the junction to show that the junction can be rectifying or ohmic, depending on the relative magnitudes of $\phi_m$ and $\phi_s$. Assume a degenerate Si with $E_F = E_C$, draw band diagrams before and after the metal contact is in contact with the semiconductor for two cases: $\phi_m < \phi_s$ and $\phi_m > \phi_s$. You must label $E_b$, $E_c$, $E_v$, and the amount of band bending clearly in your graphs. (20pts)

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3. Assume that the $E$ vs. $k$ relationship for electrons in the conduction band of a hypothetical n-type semiconductor can be approximated by:

$$E = ak^2$$

The cyclotron resonance for electrons at a magnetic field of $B = 0.1T$ is found to occur at an angular rotation frequency of $\omega_c = \frac{eB}{m} = 1.6 \times 10^{11} \text{rad/s}$. 

a) Find the value of $a$. ($\hbar = 1.0 \times 10^{-34} J \cdot s$, $e = 1.6 \times 10^{-19} C$) (20pts)

b) Estimate the donor density, $N_D$, given that the Hall coefficient at room temperature is $R_H \approx -\frac{1}{nq} = -6.25 \times 10^{-6} m^3 / C$ (10pts)