1. (20 points)
Consider a transmission line consisting of two infinitely long perfect conductors of arbitrary but consistent cross-section and consistent separation in a lossless dielectric, as shown below.

1a. (5 points)
Explain (using appropriate equations) how Maxwell's Equations can be applied to this case to determine the characteristic impedance of the TEM mode on this line.

1b. (10 points)
For the specific case of a lossless coaxial transmission line, as shown below in cross-section, apply the method you described in part (a) to determine the characteristic impedance of the TEM mode. The radius of the inner conductor is a and the outer conductor fills the annulus between b and c.

1c. (5 points)
The lossless transmission lines discussed in parts (a) and (b) support TEM waves. Suppose that the dielectric of the transmission is now slightly lossy. Does the transmission line still support a true TEM mode? Justify your answer.
2. (40 points)
Consider a parallel plate waveguide as shown below having a wave propagating in the z-direction.

![Diagram of a parallel plate waveguide](image)

In a guide that is constructed of perfectly conducting plates, the waveguide will support a TEM wave as the fundamental mode. However if the guiding plates are not perfect conductors but only good conductors, the fundamental mode will no longer be a true TEM wave; although the TEM mode of an ideal guide is a very good approximation to the actual field distribution.

2a (20 points)
Assuming a wave propagating in the fundamental mode, use the TEM approximation to determine the average power loss per unit length and width \( b \) in a real waveguide having plates of conductivity \( \sigma \) which are thick compared to the skin depth \( \delta \).

2b (10 points)
Determine the attenuation constant, \( \alpha \), for this wave.

2c (5 points)
Determine the \( z \)-component of the electric field of this wave at the surface of the lower conductor.

2d (5 points)
Determine the \( z \)-component of the electric field of this wave at the center of the waveguide.
3 (40 points)
Consider the case of a parallel plate capacitor composed of two closely spaced perfectly conducting disks of radius \(a\) and spacing \(d\), as shown in the figures below.

![Figure 3a](image)

![Figure 3b](image)

From our basic physics course we know that if a D.C. voltage is applied, as in Figure 3a, and if fringing is neglected, the electric field within the capacitor is uniform and directed normal to the surface of the plates (in the \(z\)-direction).

When an A.C. voltage is applied to the same capacitor, as shown in Figure 3b, it is often assumed that the field distribution is the same as in the static case, but now the field is modulated by \(\sin(\omega t)\), where \(\omega\) is the frequency of excitation.

3a (10 points)
Show that the assumed solution, \(\overline{E} = E_0 \sin(\omega t) \hat{z}\), violates Maxwell's Equations even when all edge effects are not considered (ignored).

3b (5 points)
Provide a brief description of the physical reason for this apparent paradox.

3c (5 Points)
Without attempting to solve the problem, provide an expression for the correct functional form to describe the electric fields within the parallel plate capacitor driven by a sinusoidal voltage.

3d (20 Points)
Develop the scalar second order differential equation which must be solved to find the actual electric field. Reduce the equation to standard form, but you do not have to solve.
POSSIBLY USEFUL DEFINITIONS AND EQUATIONS

Bessel Differential Equation:

$$\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} + \left(1 - \frac{n^2}{r^2}\right)f = 0.$$  

Gradient

cylindrical

$$\nabla \mathbf{V} = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \phi} \hat{a}_\phi + \frac{1}{r} \frac{\partial V}{\partial z} \hat{a}_z$$  
spherical

$$\nabla \mathbf{V} = \frac{\partial V}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{a}_\theta + \frac{1}{r \sin \theta} \frac{\partial V}{\partial \phi} \hat{a}_\phi$$

Divergence

cylindrical

$$\nabla \cdot \mathbf{D} = \frac{1}{r} \frac{\partial (rD_r)}{\partial r} + \frac{1}{r} \frac{\partial D_\phi}{\partial \phi} + \frac{1}{r} \frac{\partial D_z}{\partial z}$$  
spherical

$$\nabla \cdot \mathbf{D} = \frac{1}{r^2} \frac{\partial (r^2 D_r)}{\partial r} + \frac{1}{r \sin \theta} \frac{\partial (D_\theta \sin \theta)}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial D_\phi}{\partial \phi}$$

Curl

cylindrical

$$\nabla \times \mathbf{H} = \left( \frac{1}{r} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \hat{a}_r + \left( \frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} \right) \hat{a}_\phi + \frac{1}{r} \left[ \frac{\partial (rH_\phi)}{\partial r} - \frac{\partial H_r}{\partial \phi} \right] \hat{a}_z$$  
spherical

$$\nabla \times \mathbf{H} = \frac{1}{r \sin \theta} \left[ \frac{\partial (H_\theta \sin \theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi} \right] \hat{a}_r + \frac{1}{r} \left[ \frac{1}{\sin \theta} \frac{\partial H_r}{\partial \phi} - \frac{\partial (rH_\phi)}{\partial r} \right] \hat{a}_\phi$$

Laplacian

cartesian

$$\nabla^2 \mathbf{V} = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$  
cylindrical

$$\nabla^2 \mathbf{V} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$  
spherical

$$\nabla^2 \mathbf{V} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 V}{\partial \phi^2}$$

Constants

$$\varepsilon_0 = 8.854 \times 10^{-12} \text{ F/m} = \frac{1}{36\pi} \times 10^{-9} \text{ F/m}, \quad \mu_0 = 4 \pi \times 10^{-7} \text{ H/m}.$$