**Problem 1.** Suppose the baseband signal \( s(t) = b_0[p_T(t) - 3p_T(t - T)] \), where \( b_0 \) takes on values from the set \( \{-1, +1\} \) with equal probability, is received in the presence of additive white Gaussian noise (AWGN) \( n(t) \) with two-sided power spectral density \( N_0/2 \). The function \( p_T(t) \) is defined by \( p_T(t) = 1 \) for \( 0 \leq t < T \), and \( p_T(t) = 0 \), elsewhere. The receiver is shown below.

![Diagram of receiver](diagram.png)

**Figure 1:** Receiver used for Problem (1) and Problem (3).

The sum of the signal \( s(t) \) and noise \( n(t) \) passes through a filter with impulse response given by \( h(t) = p_T(t) \), and the output of the filter is sampled at time \( T \) to produce a decision statistic \( r(T) \). If \( r(T) \geq 0 \), the receiver decides that \( b_0 = +1 \), and if \( r(T) < 0 \), the receiver decides that \( b_0 = -1 \).

(a) (15 pts.) Find the average probability of error for the detection of \( b_0 \). (When necessary, express answers in terms of \( \Phi(x) \), the cumulative distribution function of a zero-mean, unit-variance Gaussian random variable.)

(b) (15 pts.) Suggest an alternative structure in which the calculation of the decision statistic in Figure 1 is modified so that the decision statistic is now a weighted sum of two samples of the signal \( r(t) \) taken at different times. Pick and identify the sampling times and weighting coefficients to minimize the average probability of error for the detection of \( b_0 \).

(c) (15 pts.) Find the average probability of error for the detection of \( b_0 \) when the optimized system that you are asked to design in Part (b) is utilized.

**Problem 2.** (20 points) Suppose that only AWGN with two-sided power spectral density given by \( 10^{-6}W/Hz \) is applied to the input of a linear time-invariant filter. The power spectral density of the output of that linear time-invariant filter is measured as \( 16[\sin(4\pi f)]/(4\pi f)^2 \), where \( f \) is the frequency in Hz. Find the impulse response of the filter as a function of \( (t - t_d) \), where \( t_d \) indicates an unknown delay. (You may wish to recall that the Fourier transform of \( p_T(t + T/2) = T[\sin(\pi f T)]/(\pi f T) \).
Problem 3. Now the situation described in Problem (1) is modified so that the received signal is the infinite data stream given by \( s(t) = \sum_{k=-\infty}^{\infty} b_k p_T(t - kT) \), where \( k \) is an integer and \( b_k \) takes on values from the set \( \{-1, +1\} \) with equal probability, plus a delayed replica of that signal. This results from the added complication of multipath, which causes a second reception of the signal that is delayed by \( T/3 \) and has half the amplitude, that is, the signal at the input to the receiver is now \( s(t) + (1/2)s(t - T/3) \).

(a) (20 pts.) Find the average probability of error for the detection of \( b_0 \), accounting for intersymbol interference.

(b) (15 pts.) Suppose that the second reception of the signal that is delayed by \( T/3 \) is now found to have an amplitude \( \alpha \) that is close to 1. Suggest an alternative structure in which the decision statistic is a weighted sum of two samples of the signal \( \hat{r}(t) \) taken at different times. Explain how you would identify the sampling times and weighting coefficients to make the average probability of error for the detection of \( b_0 \) small. Explain whether the approach that you propose is guaranteed to minimize the average error probability.