Part 1: Projection Tomography Principles [40 points]

(A) (15 pts) Assume an object having support on the intervals \([-x_0, +x_0]\) and \([-y_0, +y_0]\) is to be imaged using projection tomography. Write the mathematical relationship between the projection, \(p_b(r)\) computed using observed signal intensities and the linear attenuation coefficient, \(\mu(x, y)\). (Hint: This requires a transformation of co-ordinates.)

For parts (B)–(E), assume parallel-ray geometry.

(B) (5 pts) Sketch the sinogram associated with \(\delta(x, y)\).

(C) (5 pts) Sketch the sinogram associated with \(\delta(x - 1, y)\).

(D) (5 pts) Now sketch the sinogram obtained for the case in which our point object has been scanned at the origin for \(\theta \in [0, \frac{\pi}{2}]\), but has moved to (1,0) for \(\theta \in [\frac{\pi}{2}, \pi]\).

(E) (10 pts) Show that the sinogram of part (D) cannot be the Radon transform of any object. Note that this property holds true for any observation involving motion.

Part 2: Resonance Tomography Principles (NMRI) [60 points]

(A) (10 pts) Write the Larmor Equation. Define all terms.

(B) (10 pts) When sampling in k-space, how are the field-of-view (FOV) and spatial resolution (i.e., voxel size) of the ultimate image determined?

(C) (20 pts) Briefly describe the physical meaning of each of the \(T_1\) and \(T_2\). Include in your answer some examples of the properties of a material that may lead to a long or short value of each.

(D) (10 pts) What is the \(T_2^*\) time-constant? and how does this affect efforts to quantify \(T_1\) and/or \(T_2\)?

(E) (10 pts) Fill in (i)-(v) for the following table:

<table>
<thead>
<tr>
<th>Weighting</th>
<th>TR</th>
<th>TE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proton Density (PDw)</td>
<td>(i)</td>
<td>short TE ((\ll T_2))</td>
</tr>
<tr>
<td>((\text{ii}))</td>
<td>TR (\approx \frac{T_2}{2})</td>
<td>(iii)</td>
</tr>
<tr>
<td>(T_2) (T2w)</td>
<td>(iv)</td>
<td>(v)</td>
</tr>
</tbody>
</table>