1) The stator voltage expressions of a 3-phase wound-rotor synchronous machine with $L_q = L_d$ can be expressed in the rotor frame of reference as:

$$v_{qs}^r = r_s i_{qs}^r + \omega \lambda_{ds}^r + p \lambda_{qs}^r$$

$$v_{ds}^r = r_s i_{ds}^r - \omega \lambda_{qs}^r + p \lambda_{ds}^r$$

$$v_{fd} = r_{fj} i_{fd} + p \lambda_{fd}$$

$$v_{kq} = r_{kq} i_{kq} + p \lambda_{kq}$$

$$v_{kd} = r_{kd} i_{kd} + p \lambda_{kd}$$

a) Using these, along with the necessary relationships between flux linkages and currents, derive the steady-state expression

$$\tilde{V}_{as} = (r_s + j\omega L_q) \tilde{i}_{as} + \tilde{E}_{as}$$

Show all steps in the derivation. Make sure to show the relationship between $\tilde{E}_{as}$, the field winding current, the angle $\delta$, and the frequency of stator excitation. (21 $\frac{1}{3}$ pts)

b) Answer the following True/False Explain your reasoning for full credit. (12 pts)

a) If $i_{kq} = 0$, then $\lambda_{kq} = 0$.

b) If $\delta = 0$, $i_{qs}^r = i_{ds}^r = 0$.

c) If $\dot{i}_{kq} = 0$, $\dot{i}_{kd} = 0$.

2) The flux-linkage-versus-current relationship of a 3-phase, 4-pole permanent-magnet synchronous machine in the rotor frame of reference has the form:

$$\lambda_{qs}^r = L_q i_{qs}^r$$

$$\lambda_{ds}^r = L_d i_{ds}^r - \lambda_m$$

Draw a cross sectional view of a machine that could be modeled using these relationships. Show the phase-a winding, the rotor, the magnets, the $q$- and $d$-axis, and a rotor position angle that indicates the relative position between the stator and rotor. Express $T_e$ in terms of $i_{qs}^r$ and $i_{ds}^r$ for this machine. (33 $\frac{1}{3}$ pts)
3) A 6-pole induction machine is operated as a motor with balanced excitation and
\[ \lambda_{dr}^s = 0.1 \cos(377t), \lambda_{dr}^r = 0.1 \cos(10t). \]

a) Fill in the following table. (20 pts)

<table>
<thead>
<tr>
<th></th>
<th>actual (abc) variables</th>
<th>arbitrary reference frame</th>
<th>‘2’ reference frame - (\omega = 2) rad/s</th>
<th>synchronous reference frame - (\omega = \omega_e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>frequency of stator currents</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>frequency of rotor currents</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Determine the speed and direction of the stator and rotor MMF relative to a person a) on the stator and b) on the rotor. (13 \(\frac{1}{3}\) pts)