1. (25 Points) Let \( X, Y \) and \( Z \) be three jointly distributed random variables with joint pdf
\[
f_{XYZ}(x, y, z) = \frac{3z^2}{7\sqrt{2\pi}} e^{-zy} \exp \left[ -\frac{1}{2} \left( \frac{x - y}{z} \right)^2 \right] \cdot 1_{[0, \infty)}(y) \cdot 1_{[1, 2]}(z).
\]
(a) Find the joint probability density function \( f_{YZ}(y, z) \).
(b) Find the conditional probability density function \( f_X(x|y, z) \).
(c) Find the probability density function \( f_Z(z) \).
(d) Find the conditional probability density function \( f_Y(y|z) \).
(e) Find the conditional probability density function \( f_{XY}(x, y|z) \).

2. (25 Points) Show that if a continuous-time Gaussian random process \( X(t) \) is wide-sense stationary, it is also strict-sense stationary.

3. (25 Points) Show that the sum of two jointly distributed Gaussian random variables that are not necessarily statistically independent is a Gaussian random variable.

4. (25 Points) Assume that \( X(t) \) is a zero-mean continuous-time Gaussian white noise process with autocorrelation function
\[
R_{XX}(t_1, t_2) = \delta(t_1 - t_2).
\]
Let \( Y(t) \) be a new random process obtained by passing \( X(t) \) through a linear time-invariant system with impulse response \( h(t) \) whose Fourier transform \( H(\omega) \) has the ideal low-pass characteristic
\[
H(\omega) = \begin{cases} 1, & \text{for } |\omega| \leq \Omega, \\ 0, & \text{elsewhere,} \end{cases}
\]
where \( \Omega > 0 \).
(a) Find the mean of \( Y(t) \).
(b) Find the autocorrelation function of \( Y(t) \).
(c) Find the joint pdf of \( Y(t_1) \) and \( Y(t_2) \) for any two arbitrary sample times \( t_1 \) and \( t_2 \).
(d) what is the minimum time difference \( t_1 - t_2 \) such that \( Y(t_1) \) and \( Y(t_2) \) are statistically independent?

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