In his seminal paper published in Annalen der Physik in 1907 (vol. 23, p. 846 - 866), J. Zenneck was the first to analyze a solution of Maxwell's equations that had a “surface wave” property. This so-called Zenneck wave is simply a wave solution to Maxwell's equations in the presence of a planar boundary that separates free space from a half space with a finite conductivity. The amplitude of this wave decays exponentially in the directions perpendicular to the boundary (with differing decay constants “above” and “below” the surface - see Fig. 2). Surface plasmons that propagate along a metal-dielectric interface and hold the promise of the emerging field of “plasmonics” to revolutionize modern optoelectronics, represent a special case of the Zenneck wave.

Figure 1: Jonathan A. W. Zenneck (April 15, 1871 - April 8, 1959)

Problem 1. 35 points
Treating the electromagnetic response of the metal as arising primarily from free electrons with effective mass $m_{\text{eff}}$ and density (i.e., number of electrons per unit volume) $n$, and neglecting their scattering, determine the frequency range where the plasmon can exist. Assume that the permittivity of the dielectric medium above the metal (see Fig. 2) $\varepsilon_d$ is also known.

Problem 2. 35 points
Under the assumptions of Problem 1, calculate
(i) the electromagnetic field of the surface plasmon wave $\{E(x, y, z; t), B(x, y, z; t)\}$ at frequency $\omega$.
(ii) its wavenumber $k$ as a function of frequency. Calculate the corresponding functional dependence, and plot it in $\omega$ vs. $k$ coordinates.

Figure 2:
Problem 3. 10 points
Explain (qualitatively) how the result of Problem 2 would change when electron scattering is taken into account.

Problem 4. 20 points
Assuming the electron scattering time $\tau$ is known, calculate the propagation distance of the plasmon as a function of frequency. (You can define it as the distance corresponding to the decrease of the plasmon field by $1/e$). Assume that $\epsilon_d$, $m_{\text{eff}}$, and $n$ are known.