1. (20 points)
Consider a spherical conductor of radius 5 cm centered at the origin. Outside the sphere is vacuum. The electric field at a point \( P \) located a distance \( r \) from the origin is given by \( E = \hat{r} \frac{1}{4\pi r^2} \) (V/m) for 5 cm < \( r \) < 6 cm, where \( \hat{r} \) is the unit vector from the origin pointing toward the point. What is the total charge \( Q \) on the surface of this spherical conductor? Please give your reasoning. (Note: we unfortunately do not know the electric field for \( r > 6 \text{cm} \)).

2. (30 points)
The circular disk shown below has radius \( a \) and infinitesimally small thickness, \( d \) (i.e., \( d \to 0 \)). Therefore we can define a surface charge density \( \rho_s \) with a unit of Coulomb/m\(^2\). The surface charge density is uniform across the disk. Point \( O \) is the center of the disk and line \( PO \) is perpendicular to the disk. The distance between \( P \) and \( O \) is \( h \) (assume \( h > 0 \)).

(a) (15 points) Find the electric potential \( V \) at point \( P \) in terms of \( h \), \( a \), \( \rho_s \) and \( \varepsilon_0 \);
(b) (15 points) Find the electric field \( E \) at point \( P \) in terms of \( h \), \( a \), \( \rho_s \) and \( \varepsilon_0 \).
3. (30 points)
A coaxial line shown below is infinitely-long. For simplicity, we assume the current \( I \) flows along the \( z \) direction on the surface of an inner cylinder of radius \( a \), and the current of the same magnitude, \( I \), flows against the \( z \) direction on the surface of an outer cylinder of radius \( b \). We assume that the coaxial line forms a closed-loop circuit, except that the loop is connected at infinity.

![Diagram of coaxial line](image)

(a) (15 points) Derive the magnetic field, \( \mathbf{H} \) at a generic point \( P(\rho, \phi, z) \)
(b) (15 points) Find the inductance per unit length of the line, assuming the medium between the two cylinders is vacuum.

4. (20 points)
An infinitely long conducting cylinder of radius \( a \) is placed a distance \( d \) above an infinite, grounded plane, as shown below. The linear charge density on the cylinder is \( \rho_l \) (Coulomb/m).

![Diagram of cylinder above plane](image)

Describe the analytical procedure (no experimental procedures) you plan to use to find the capacitance per unit length for this system. Mathematical derivations are not required.
Maxwell's Equations:
\[ \nabla \cdot \mathbf{D} = \rho, \quad \oint_S \mathbf{D} \cdot d\mathbf{S} = Q_{\text{enc}} \]
\[ \nabla \cdot \mathbf{B} = 0 \quad \oint_S \mathbf{B} \cdot d\mathbf{S} = 0 \]
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \]
\[ \oint_c \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt} \oint_S \mathbf{B} \cdot d\mathbf{S} \]
\[ \oint_c \mathbf{H} \cdot d\mathbf{l} = I_{\text{enc}} + \frac{d}{dt} \oint_S \mathbf{D} \cdot d\mathbf{S} \]

Poynting's Theorem:
\[ \nabla \cdot (\mathbf{E} \times \mathbf{H}) = -\frac{\partial}{\partial t} \left( \frac{1}{2} \mu \mathbf{H} \cdot \mathbf{H} \right) - \frac{\partial}{\partial t} \left( \frac{1}{2} \varepsilon \mathbf{E} \cdot \mathbf{E} \right) - \mathbf{J} \cdot \mathbf{E} \]

Potentially useful vector algebra
\[ \nabla \times \mathbf{A} = \hat{x} \left( \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y} \right) + \hat{y} \left( \frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} \right) + \hat{z} \left( \frac{\partial A_x}{\partial y} - \frac{\partial A_y}{\partial x} \right) \]
\[ \nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \]
\[ \nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z} \]

Potentially useful integral identities
\[ \int \frac{dx}{x} = \ln x \]
\[ \int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( x + \sqrt{x^2 + a^2} \right) \]
\[ \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} \]
\[ \int \frac{dx}{(x^2 + a^2)^{3/2}} = \frac{x}{a^2 \sqrt{x^2 + a^2}} \]
\[ \int \sin^2 x = \frac{x}{2} - \frac{1}{4} \sin 2x \]
\[ \int \sin^3 x = \frac{\cos x}{3} - \cos x \]
\[ \int \cosh^2 x = \frac{1}{2} [-x + \sinh x \cosh x] \]
\[ \int \frac{dx}{x^{3/2}} = \frac{1}{\sqrt{x} + a^2} \]
\[ \int \frac{dxdx}{\sqrt{x^2 + a^2}} = \frac{1}{2} \ln \left( x^2 + a^2 \right) \]
\[ \int \frac{dxdx}{(x^2 + a^2)^{3/2}} = -\frac{1}{\sqrt{x^2 + a^2}} \]
\[ \int \cos^2 x = \frac{x}{2} + \frac{1}{4} \sin 2x \]
\[ \int \cos^3 x = -\frac{\sin^3 x}{3} + \sin x \]
\[ \int \cosh^3 x = \frac{1}{2} [x + \sinh x \cosh x] \]