Q1 (25 points) Consider the following recursion for printing a sequence of integers:

```c
void Seq(int n)
{
    if (n == 1)
        write("1 ");
    else
        Seq(n-1);
        write(n);
        Seq(n-1);
        for i ← 1 to (2*log n) \{**Assume 2*log n is an integer**\}
            write( i);
}
```

(a) (10 points) Write a recurrence equation that describes the number of numerals of output generated by Seq(n). Include the base case.

(b) (15 points) Find the complexity of above recursive code using the Master Theorem if applicable. Otherwise use an alternate method to find the complexity using \( \Theta \)-notation.

Q 2. (20 points) Prove or disprove that a heap with \( n \) elements can be converted into a binary search tree in \( O(n) \) time (using only comparisons).

Q 3. (30 points) The State of Indiana is divided by the Wabash River into the east and west banks. There are \( n \) towns along each of the two banks. Each town on the east bank (Say \( E_i \) thru \( E_n \)) has its unique friend town on the west bank (say \( W_i \) thru \( W_n \)). No two towns have the same friend. Each pair of friend towns would like to have a ship line connecting them. They applied for ship-line permission to the government. Because it is often foggy on Wabash River the government decided to prohibit intersection of ship lines (if two lines intersect there is a high probability of ship crash).

(a) (10 points) What is the complexity of a brute-force algorithm if you need to identify the maximum number of permissible ship lines? Provide reasoning for your answer.

(b) (20 points) Can this problem be solved using dynamic programming? If yes, then prove the problem has an optimal substructure property and define a recursive formulation of the problem. What is the complexity of your dynamic programming solution?

Q4. (25 points) Given a list of courses, a list of conflicts between pairs of courses (a pair of courses have a conflict if a student has registered for both courses), and an integer \( k \), prove that determining an exam schedule consisting of \( k \) dates such that there are no
conflicts among courses which have conflicts on the same date is NP-complete. (Hint: K-Graph-Color – a problem that determines if a graph can be colored with k-colors such that no two adjacent vertices have the same color – is a known NP-complete problem.)