Problem 1. (40 points) The real baseband received signal \( r(t) \) is given by

\[
r(t) = s_i(t) + n(t); i = 0, 1,
\]

where \( \Pr\{i = 0\} = \Pr\{i = 1\} = 1/2 \), and \( n(t) \) is an additive white Gaussian noise process with two-sided power spectral density \( N_0/2 \). The two real signals \( s_0(t) \) and \( s_1(t) \) are illustrated in the following figure, and are equal to zero outside the interval \([0, T]\).

Suppose that the receiver structure is given as shown below. The received signal is processed with a linear time-invariant filter with impulse response \( h(t) \), and the output of that filter is sampled at time \( t = T \) to produce the statistic \( z_i \), which is compared with a threshold \( \gamma \). If \( z_i > \gamma \), \( i = 0 \) is decided. If \( z_i < \gamma \), \( i = 1 \) is decided.

(a) First, suppose \( h(t) = p_T(t) \), that is, \( h(t) = 1 \) for \( 0 \leq t < T \), and \( h(t) = 0 \), elsewhere. Find the minimax threshold, that is, the threshold that minimizes the maximum of the probability of error given that \( s_0(t) \) is sent and the probability of error given that \( s_1(t) \) is sent. Also, find the corresponding average probability of decision error in terms of \( N_0 \) and \( T \).
(b) Now, suppose the impulse response $h(t)$ can be adjusted. Find and sketch the impulse response of the matched filter for this signal set. Label key values in your sketch.

(c) Find the average error probability when the matched filter and the minimax threshold for the matched filter are used in terms of $N_0$ and $T$.

(d) Now suppose that the problem stated in Part (a) remains the same except for the manner in which the statistic $z_i$ is formed. Now, the output of the filter is sampled twice, once at $T/2$ and once at $T$, and then the samples are added to form the new statistic $z_i$. Compute how the answer to Part (a) changes.

**Problem 2.** (40 points) Thermal noise with two-sided spectral density $N_0/2 = 4 \times 10^{-6}$ W/Hz is the input to a linear time-invariant filter with impulse response $h(t) = 5 \exp(-2t)u(t)$, where $u(t)$ is the unit step function defined as $u(t) = 1$, for $t \geq 0$, and $u(t) = 0$, elsewhere.

(a) Find the power spectral density of the noise at the output of the filter.

(b) What is the total power at the output of the filter that results from the noise?

(c) Now, the signal $3\cos(2\pi 10t)$, for $-\infty < t < \infty$, is applied to the input of the filter. What is the total power at the output of the filter that results from the (steady state) signal, ignoring the effects of noise? (You may express your answer in terms of $\pi$.)

(d) At the output of the filter what is the (power) signal-to-noise ratio (SNR), defined as the ratio of signal power to noise power, when both signal and noise are applied at the input to the filter? (You may express your answer in terms of $\pi$.)
Problem 3. (20 points) In a short paragraph describe the impact of a multipath channel on the strength of a received radio-frequency signal. Define the terms delay spread and coherence bandwidth. Explain any relationships between these terms.