Problem 1. (34pt)
Let \( h(m, n) \) be a low pass filter with an impulse response of

\[
h(m, n) = \begin{cases} 
1/9 & |m| \leq 1 \text{ and } |n| \leq 1 \\
0 & \text{otherwise}
\end{cases}
\]

Then define the filter impulse response

\[
g(m, n) = \delta(m, n) + \lambda(\delta(m, n) - h(m, n))
\]

where \( \lambda \) is a parameter which can be adjusted to achieve a desired result and \( \delta(m, n) \) is a discrete-space impulse.

a) (12pt) Calculate an analytical expression for \( H(e^{j\mu}, e^{j\nu}) \) the DSFT of \( h(m, n) \).

b) (11pt) Calculate an analytical expression for \( G(e^{j\mu}, e^{j\nu}) \) the DSFT of \( g(m, n) \).

c) (11pt) What is the effect of increasing and decreasing \( \lambda \) in the range \( \lambda > 0 \).
Problem 2. (33pt)
Consider the 2D difference equation

\[ y(m, n) = bx(m, n) + ay(m - 1, n) + ay(m, n - 1) - a^2 y(m - 1, n - 1) \]

where \( b \in \mathbb{R} \) and \( a \in (-1, 1) \) are two constants, and \( Y(z_1, z_2) \) and \( X(z_1, z_2) \) are the 2D Z-transforms of \( y(m, n) \) and \( x(m, n) \) respectively.

a) (11pt) Calculate \( H(z_1, z_2) = \frac{Y(z_1, z_2)}{X(z_1, z_2)} \), the 2D transfer function of the causal system. Make sure to express your result in factored form.

b) (11pt) Calculate, \( h(m, n) \), the impulse response of the system with transfer function \( H(z_1, z_2) \).

c) (11pt) In an application, \( x(m, n) \) is an input image, and \( y(m, n) \) is an output filtered image. Specify a relationship between \( a \) and \( b \) so that the average values of the input and output images remain the same.
Problem 3. (33pt)
Consider an achromatic image, \( X(m, n) \), which is gamma corrected with \( \gamma = 2.2 \) using simple power-law gamma correction.

a) (11pt) What is the formula used to convert \( X(m, n) \) to \( I(m, n) \), where \( I(m, n) \) has units proportional to energy (i.e. number of photons)?

b) (11pt) If it is necessary to represent the image with 8 bit quantization, is it better to quantize \( X(m, n) \) or \( I(m, n) \)? Why?

c) (11pt) If you need to accurately view the image \( X(m, n) \) on a display with a gamma of 1.8, what must you do? Be specific.