1. (35 pts.) A communication system transmits one of three signals:

\[ s_0(t) = K p_T(t) \sin(\omega_c t), \]

\[ s_1(t) = 0, \text{ and} \]

\[ s_2(t) = -K p_T(t) \sin(\omega_c t), \]

where \( K \) is a positive number and \( p_T(t) = 1 \) for \( 0 \leq t < T \), and \( p_T(t) = 0 \), elsewhere.

over an additive white Gaussian noise channel with spectral density \( N_0/2 \) at a carrier frequency \( \omega_c \) satisfying \( \omega_c > 1/T \). Let \( W(t) \) denote the received signal \( (W(t) = X(t) + s_i(t) \) for \( i=0,1,2 \), where \( X(t) \) is the noise process). The receiver computes the quantity

\[ U = \int_0^T W(t) \sin(\omega_c t) dt. \]

\( U \) is compared with a positive threshold \( \gamma \) and a threshold \( -\gamma \). If \( U > \gamma \), the decision is made that \( s_0(t) \) was sent. If \( U < -\gamma \), the decision is made that \( s_2(t) \) was sent. If \( -\gamma < U < \gamma \), the decision is made in favor of \( s_1(t) \).

(a) Determine the three conditional probabilities of error: \( P_{e,0} = \) probability of error given \( s_0 \) sent, \( P_{e,1} = \) probability of error given \( s_1 \) sent, and \( P_{e,2} = \) probability of error given \( s_2 \) sent. Express your answer in terms of the Q function defined by

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du. \]

(b) Determine the average error probability assuming that all three signals are transmitted with equal probability.

2. (30 pts.) A binary communication system transmits one of two equally likely signals \( s_0(t) \) and \( s_1(t) \) of duration \( T \) given for \( i=0,1 \) by

\[ s_i(t) = \mathbb{B}(-1)^i p_T(t) \cos(\omega_0 t). \]

The noise \( n(t) \) in the system is white Gaussian noise with power spectral density \( N_0/2 \). The receiver shown below is used to demodulate the signal. However, as shown, the phase of the received signal is not known completely accurately. In fact, there is a discrepancy of \( \theta \) radians between the received signal (in the absence of noise) and the local reference signal. Determine the error probability at the output of the demodulator as a function of \( \theta \). (Ignore double frequency terms and express your answer in terms of the Q function.)
3. (35 pts.) The receiver in a QPSK communications system consists of two parallel branches as shown below.

\[ s(t) \rightarrow g(t) + n(t) \rightarrow \cos(\omega_c t) \times h(t) \rightarrow i = T \]

\[ U(iT) \]

\[ U > 0 \] dec. \[ U_{i-1} = +1 \]

\[ U < 0 \] dec. \[ U_{i-1} = -1 \]

\[ t = T \rightarrow h(t) \]

\[ V(iT) \]

\[ V > 0 \] dec. \[ V_{i-1} = +1 \]

\[ V < 0 \] dec. \[ V_{i-1} = -1 \]

\[ \sin(\omega_c t) \]

Each branch has a multiplier, a linear time-invariant filter, a sampler, and a threshold device. The baseband impulse response \( h(t) \) (with bandwidth much smaller than the carrier frequency \( \omega_c \)) satisfies

\[ \int_{-\infty}^{\infty} h^2(t) dt = 9 \]

The channel is an AWGN (additive white Gaussian noise) channel and \( \omega_c \gg 1/T \). The white Gaussian noise process \( n(t) \) has spectral density \( N_0/2 = 2 \). The transmitted signal \( s(t) \) is given by

\[ s(t) = A u(t) \cos \omega_c t + A v(t) \sin \omega_c t. \]

The impulse response \( g(t) \) is unknown. It is known that if \( n(t) = 0 \), \( u(t) = 0 \), and \( v(t) = p_T(t) \), then

\[ V(T) = 5, \quad V(2T) = 2, \quad U(T) = 1, \]

and all other values of \( V(kT) \) and \( U(nT) \) are zero. Similarly, if \( n(t) = 0 \), \( u(t) = p_T(t) \), and \( v(t) = 0 \), it is observed that

\[ U(T) = 5, \quad U(2T) = 3, \quad V(T) = 1, \]

and all other values of \( U(nT) \) and \( V(kT) \) are zero.

Give an expression for the average probability of error for each of the two branches of the receiver if \( u(t) \) and \( v(t) \) are now statistically independent, infinite sequences of unit amplitude, positive and negative, rectangular pulses of duration \( T \). That is,

\[ u(t) = \sum_{n=-\infty}^{\infty} U_n p_T(t - nT) \text{ and } v(t) = \sum_{k=-\infty}^{\infty} V_k p_T(t - kT), \]

where \( \{ U_n \} \) and \( \{ V_k \} \) are statistically independent sequences of independent random variables with \( P(U_n = +1) = P(U_n = -1) = P(V_k = +1) = P(V_k = -1) = 1/2 \). Express your answer in terms of the \( Q \) function. (Please do not leave this problem blank, but at least calculate the variance of the decision statistics.)