Problem 1. [70 pts] Consider the discrete-time system described by the difference equation below.

\[ y[n] = x[n] - x[n - 4] \]  
(1)

(a) (10 pts) Plot the magnitude \(|H(e^{j\omega})|\) of the frequency response of this system, equal to the Discrete Time Fourier Transform (DTFT) \(H(e^{j\omega})\) of the impulse response \(h[n]\), over \(-\pi < \omega < \pi\). Show as much detail as possible.

For all parts, a discrete-time (DT) random process \(x[n]\) is input to the system above. The DT random process is obtained by sampling a continuous time signal \(x_a(t)\) whose autocorrelation function, \(r_{x_a x_a}(\tau) = \mathcal{E}\{x_a(t)x_a(t - \tau)\}\), is given, where \(\mathcal{E}\{\cdot\}\) represents expected value operator in a statistical sense.

For each part below, you must do EACH of the following THREE steps.

(i) Plot the spectral density \(S_{x_a}(e^{j\omega})\) of the input sampled signal \(x[n]\).

(ii) Plot the spectral density \(S_{y[n]}(e^{j\omega})\) of the corresponding output signal \(y[n]\).

(iii) Determine the numerical value of \(\mathcal{E}\{y^2[n]\}\).

You MUST show all work to receive full credit.

(b) (12 pts) \(x[n] = x_a(nT_a)\) where \(T_a = \frac{2\pi}{40}\) and \(r_{x_a x_a}(\tau) = \frac{2\pi}{40} \left\{ \frac{\sin(10\tau)}{\pi\tau} \right\}\).  

(c) (12 pts) \(x[n] = x_a(nT_a)\) where \(T_a = \frac{2\pi}{40}\) and \(r_{x_a x_a}(\tau) = \frac{2\pi}{40} \left\{ \frac{\sin(5\tau)}{\pi\tau} \right\}\) \(\cos(15\tau)\).

(d) (12 pts) \(x[n] = x_a(nT_a)\) where \(T_a = \frac{2\pi}{20}\) and \(r_{x_a x_a}(\tau) = \frac{2\pi}{20} \left\{ \frac{\sin(10\tau)}{\pi\tau} \right\}\).

(e) (12 pts) \(x[n] = x_a(nT_a)\) where \(T_a = \frac{3\pi}{20}\) and \(r_{x_a x_a}(\tau) = \frac{3\pi}{20} \left\{ \frac{\sin(10\tau)}{\pi\tau} \right\}\).

(f) (12 pts) \(x[n] = x_a(nT_a)\) where \(T_a = \frac{2\pi}{40}\) \& \(r_{x_a x_a}(\tau) = 1 + \cos(10\tau) + \cos(20\tau)\).

PROBLEM 2 ON NEXT PAGE.
Problem 2. [30 pts] Consider the discrete-time LTI system described by the following difference equation.

\[ y[n] = x[n] + x[n - 1] + x[n - 2] + x[n - 3] \]  \hspace{1cm} (2)

(a) (9 pts) Plot the magnitude \( |H(e^{j\omega})| \) of the frequency response of this system equal to the DTFT \( H(e^{j\omega}) \) of the impulse response \( h[n] \) as a function of frequency over \(-\pi < \omega < \pi\). Show as much detail as possible.

(b) For \( x[n] = x_o(nT_s) \) where \( T_s = 3 \) and \( x_o(t) = u(t) - u(t - 10) \), do the following:

(i) (7 pts) Plot the magnitude \( |X(e^{j\omega})| \) of the DTFT of the sampled signal \( x[n] \) which is input to the discrete-time system described by equation (2) above.

(ii) (7 pts) Plot the magnitude \( |Y(e^{j\omega})| \) of the DTFT of the corresponding output signal \( y[n] \).

(iii) (7 pts) Determine the numerical value of \( \int_{-\pi}^{\pi} |Y(e^{j\omega})|^2 \, d\omega \). Show all work.