Quantifying the Impact of Domain Knowledge and Problem Framing on Sequential Decisions in Engineering Design

Murtuza Shergadwala  
Graduate Research Assistant  
School of Mechanical Engineering  
Purdue University, Indiana 47907  
mshergad@purdue.edu

Ilias Bilionis  
Assistant Professor  
School of Mechanical Engineering  
Purdue University, Indiana 47907  
ibilion@purdue.edu

Karthik N. Kannan  
Professor  
Krannert School of Management  
Purdue University, Indiana 47907  
kkarthik@purdue.edu

Jitesh H. Panchal  
Associate Professor  
School of Mechanical Engineering  
Purdue University, Indiana 47907  
panchal@purdue.edu

June 3, 2018

Abstract

Many decisions within engineering systems design are typically made by humans. These decisions significantly affect design outcomes, and the resources used within design processes. While decision theory is increasingly being used from a normative standpoint to develop computational methods for engineering design, there is still a significant gap in our understanding of how humans make decisions within the design process. Particularly, there is lack of knowledge about how an individual’s domain knowledge and framing of the design problem affects information acquisition decisions. To address this gap, the objective of this paper is to quantify the impact of a designer’s domain knowledge and problem framing on their information acquisition decisions and the corresponding design outcomes. The objective is achieved by (i) developing a descriptive model of information acquisition decisions, based on an optimal one-step look ahead sequential strategy, utilizing expected improvement maximization, and (ii) using the model in conjunction...
with a controlled behavioral experiment. The domain knowledge of an individual is measured in the experiment using a Concept Inventory, whereas the problem framing is controlled as a treatment variable in the experiment. A design optimization problem is framed in two different ways: a domain-specific track design problem, and a domain-independent function optimization problem. The results indicate that when the problem is framed as a domain-specific design task, the design solutions are better and individuals have a better state of knowledge about the problem, as compared to the domain-independent task. The design solutions are found to be better when individuals have a higher knowledge of the domain and they follow the modeled strategy closely.

Keywords: Sequential Information Acquisition, Decision Making, Domain Knowledge, Problem Framing
Nomenclature

$\lambda$  Confidence about the constraint boundary location

$I_0$  Initial Information

$X$  Design Space

$\mu_\lambda$  An individual’s mean prior belief about the confidence about the constraint boundary location

$\mu_b$  An individual’s mean prior belief about the location of the constraint boundary

$\mu_{\text{diff}}$  The lack of knowledge about the location of the constraint boundary

$\sigma^2$  Deviation from the modeled strategy or decision making errors

$\sigma^2_\lambda$  Variance of $\mu_\lambda$ in a Gaussian prior

$\sigma^2_b$  Variance of $\mu_b$ in a Gaussian prior

FOP  Function Optimization Problem

TDP$_{\text{CNS}}$  Track Design Problem where Constraints-are-Not-Specified

TDP$_{\text{CS}}$  Track Design Problem where Constraints-are-Specified

$\theta$  Type of an individual

$b$  Location of the constraint boundary

$E(x)$  Enjoyment objective function

$E_{\text{max}}$  Maximum Enjoyment value

$f(x)$  Design objective Function

$g(x)$  Design constraint Function

$H$  Height of the track

$S$  FCI score

$w$  Width of the valley in the track

$x$  Variable in the design space

$z$  Boolean value for constraint satisfied or not

EI  Expected Improvement

FCI  Force Concept Inventory

GP  Gaussian Process

IRT  Item Response Theory

SIADM  Sequential Information Acquisition and Decision Making
1 Introduction

Decision making is widely recognized as an integral activity within the design process [1, 2]. During the past two decades, decision-based design [1, 3] has emerged as an important research area focused on supporting a rigorous application of mathematical principles and decision theory to develop computational methods for engineering design. Existing research has focused on aspects related to decision making such as modeling preferences [4, 5], understanding deviations from rationality [6], group decision making [7], and accounting for customers’ decisions in the product design [8]. However, the emphasis of research in decision-based design has primarily been on using normative theory to make artifact decisions using a specified state of information (see Figure 1).

Much less attention has been given to descriptive theory, i.e., understanding how humans actually make decisions within the design process. As humans are an integral part of design processes, descriptive theory is essential to make better predictions about the impact of human decision making on design outcomes.

Figure 1: Illustration of past research emphasis in Decision-based Design and the focus of this paper.

Although descriptive theories of human decisions have been developed within behavioral eco-
nomics, psychology, and cognitive science, research in these fields does not address the nuances of systems engineering and design. For example, in typical engineering design processes, designers rarely make artifact decisions solely based on available information. They also perform information acquisition activities such as executing simulation models and experiments. In such activities, designers make decisions about what new information to acquire and when to stop acquiring information. Such information acquisition decisions heavily influence design outcomes and the resources utilized in the engineering design processes [9]. Therefore, there is a need for understanding how humans make information acquisition decisions within the context of engineering systems design.

Currently in design literature, to the best of our knowledge, there are no descriptive models of sequential information acquisition and decision making. Existing models of sequential decision making [10, 11, 12] do not consider a design context or the cognitive limitations of humans while making sequential decisions. For example, the authors [10, 11] assume that the individual has a finite set of choices. In the context of a design scenario, a design space can be continuous with infinite possibilities of design alternatives. A step towards descriptive modeling of such scenarios is to incorporate impact of factors such as a designer’s domain knowledge and the framing of a design problem on the design outcomes.

Factors such as a designer’s domain knowledge and the framing of a design problem affect designers’ decisions [13, 14, 15, 16]. However, there is a significant gap in our understanding of how these factors affect information acquisition decisions. For example, it is trivial to predict that an expert roller coaster designer would design better roller coasters than a novice. However, there is a lack of descriptive models that quantify the impact of a designer’s domain-specific knowledge, such as, their knowledge of dynamics on the quality of design solutions. Therefore, in this study, our objective is to quantify the impact of a designer’s domain knowledge and problem framing on their information acquisition decisions and the corresponding design outcomes.

Our approach to achieve this objective consists of two steps. First, we develop a descriptive model of a sequential information acquisition activity in an engineering design process. The model is described in Section 2. It is based on the assumptions that individuals strive to maximize their expected payoff and use the Bayesian approach to update their state of knowledge based on new information. Second, we design and execute a behavioral experiment. We utilize experimental data to estimate parameters in the model, and to test hypotheses about the impact of domain knowledge.
and problem framing on design outcomes. Before the experiment, we measure an individual’s domain knowledge using a Concept Inventory [17]. Within the experiment, we control for problem framing by presenting a mathematically identical problem in two different ways and observe the decisions made by the participants. The details of the experiment are provided in Section 3. The results are discussed in Section 4. Finally, we discuss the implications of this study, the validity of the modeling assumptions, and the avenues for future research in Section 5.

2 A Descriptive Model of Sequential Information Acquisition and Decision Making Process

Information acquisition can be broadly categorized into sequential or parallel processes [18]. In a sequential process, information is acquired in steps, and in each step, the acquired information is used to update past beliefs, resulting in a new state of knowledge at the end of that step. Hence, the information acquired in a sequential process affects the subsequent information acquisition decisions. For example, the information acquisition process is sequential when a designer decides what next experiment to conduct based on the result of previous experiments. In parallel processes, all acquired information is analyzed at the end of the process [18]. For example, the information acquisition process is parallel when a designer executes a preplanned set of experiments and analyzes the results of the entire set at the end. Within the context of engineering systems design, we recognize that both sequential and parallel information acquisition processes exist. However, in this paper we focus on modeling a single designer as a decision maker who sequentially acquires information to search for an optimal design solution.

2.1 Sequential Information Acquisition and Decision Making: An Abstraction of Design Processes

Consider a design scenario where a designer has a set of design variables $x$ that affect a design outcome $f(x)$ under constraints $g(x) \geq 0$. The designer’s objective is to achieve the best feasible design outcome. The designer does not explicitly know the mathematical relationship between the design variables and the design outcome, i.e., the function $f(x)$. However, they may know the feasibility of the design variables, i.e., the constraint function $g(x) \geq 0$, due to factors such as their
domain knowledge. In such a scenario, a designer needs to acquire information about the impact of design variables $x$ on the design outcome $f(x)$. Such information can be acquired by running (physical or computational) experiments, which incur certain cost. Consequently, they also receive information about the feasibility of the design variables $g(x)$. We assume that the designer updates their state of knowledge about both these functions after executing each experiment. Such a design scenario will be referred to as a Sequential Information Acquisition and Decision Making (SIADM) scenario.

We choose such a scenario, where $f(x)$ is unknown but $g(x)$ may be known, to decouple the impact of an individual’s domain knowledge on the knowledge of the objective $f(x)$ and the constraint function $g(x)$. In reality, designers may have knowledge about the objective function $f(x)$. We align the modeled scenario with our experiment by ensuring that the function $f(x)$ is unknown to the participants. For further details, refer to Section 3.1.

![Figure 2](http://mechanicaldesign.asmedigitalcollection.asme.org/ on 06/29/2018 Terms of Use: http://www.asme.org/about-asme/terms-of-use)
a sequence of steps, \( t = 1 \ldots T \). During each step \( t \), the decision maker chooses a set of design variables \( x_t \) to execute an experiment. Choosing a set of design variables \( x \) is referred to as *sampling*. From the experimental data, the decision maker acquires new information about \( f(x) \) and \( g(x) \), and processes it to update their state of knowledge. Then, the decision maker decides whether to continue acquiring information (or to stop experimentation). If they decide to continue acquiring information, the decision maker repeats the same set of activities as that of the previous step. If their decision is to stop experimentation, then they make artifact decisions such as selecting an alternative using the current state of knowledge at step \( t \).

2.2 Model Formulation

We begin the SIADM model formulation by making assumptions about the representation of an individual’s state of knowledge, how they update their state of knowledge, and how they make information acquisition decisions. We assume the following: A1.1) The state of knowledge of an individual is the belief of that individual. A1.2) Individuals update their state of knowledge through Bayesian updating. A2.1) The decision to choose the “next \( x \)” (refer to Figure 2) is made by maximizing the expected improvement in the objective function. A2.2) Individuals have bounded rationality [19]. A3) The decision maker stops after a fixed number of steps \( T \).

A1.1 implies that the state of knowledge of an individual is a probability distribution (referred to as a belief) assigned by the individual over the information space. By assuming A2.1 and A2.2 we imply that individuals can only estimate the impact of the information acquired in the immediate next step. In other words, we model a myopic information acquisition decision by accounting for an individual’s bounded cognitive capabilities. Assumption A3 is reasonable in scenarios where the designer has to commit to expending certain resources before conducting any experiments or there is a fixed predefined budget for experiments.

In our model, we account for the impact of domain knowledge and the framing of the design problem on the state of knowledge of an individual. We do so by utilizing a set of parameters called *type* (\( \theta \)) of an individual. The type \( \theta \) of an individual accounts for their unique characteristics due to which they have varying domain knowledge and different information acquisition strategies. By assuming A1.1 we imply that an individual’s type \( \theta \) impacts their prior beliefs with which they begin the SIADM process. While we make specific assumptions about the information acquisition
process of an individual \((A1 \text{ to } A3)\), we account for their deviation from the process through their type \(\theta\).

In the following subsections, we mathematically define two concepts: (i) the information acquired at each decision making step (Section 2.2.1) and (ii) the type of an individual \(\theta\) (Section 2.2.2). We then describe various aspects of an individual’s type \(\theta\), such as, an individual’s state of knowledge (Section 2.2.3), how they update their state of knowledge (Section 2.2.4), and how they make information acquisition decisions (Section 2.2.5).

### 2.2.1 Information Acquired at Each Step

At each decision making step, \(t = 1, 2, \ldots, T\), (refer to Figure 2) the individual samples an \(x_t\) value and receives information about:

1. The constraint feasibility, \(z_t = \begin{cases} 
1, & \text{if } g(x_t) \geq 0 \\
0, & \text{otherwise.}
\end{cases}\)

2. The value of the objective function, \(y_t = f(x_t)\), provided the constraint is satisfied, \(z_t = 1\).

The design outcome is not known, \(y_t = \emptyset\), if the constraint is not satisfied, \(z_t = 0\).

We assume that an individual begins the SIADM process at step \(t = 0\) with some initial information \(I_0\) at \(x = x_0\) about the objective function \(y_0 = f(x_0)\) and the constraint feasibility \(z\).

Thus,

\[
I_0 = \{(x_0, y_0, z_0)\}. \tag{1}
\]

The information \(I_t\) that the individual observes at the end of step \(t\) is:

\[
I_t = I_{t-1} \cup \{(x_t, y_t, z_t)\}. \tag{2}
\]

### 2.2.2 The Type of an Individual

The type \(\theta\) of an individual fully specifies (i) their prior state of knowledge and how it is represented, (ii) how they update their state of knowledge after observing \(I_t\), and (iii) how they decide what to observe at each step. Obviously, there are infinitely many modeling alternatives for items (i)-(iii).

In what follows, we have made specific modeling choices, trying to be parsimonious (to keep the
number of model parameters as small as possible), while taking into account some of the cognitive
limits of humans.

2.2.3 Modeling an Individual’s State of Knowledge

We utilize Gaussian process prior [20] to model an individual’s state of knowledge about the objective function $f(x)$. Existing studies support the findings that Gaussian Process models can capture human search process [21, 22]. A relevant study conducted by Borji and Itti [23] focuses on investigating the underlying algorithms that humans utilize to optimize an unknown 1D objective function. Their results indicate that Gaussian Process models can capture an individual’s state of knowledge about the mathematically unknown objective function $f(x)$.

We assume that prior to observing any data, the individual believes that $f(x)$ could be any sample from a Gaussian process prior [20],

$$f(x|\theta \sim \text{GP}(0, c(x, x'))),$$

with a zero mean and covariance function $c(x, x')$. The covariance function $c(x, x')$ defines the process’ behavior between any two points $x$ and $x'$. The choice of the covariance function $c(x, x')$ along with the prior beliefs that the individual has about its parameters are, in general, a part of their type $\theta$.

In this work, however, we assume that the individuals use a squared exponential covariance function:

$$c(x, x') = s^2 \exp\left\{-\frac{(x - x')^2}{2\ell^2}\right\},$$

with unspecified signal strength $s > 0$ and length scale $\ell > 0$, i.e., they assign flat priors. This choice is equivalent to the assumption that the individual believes that $f(x)$ is infinitely differentiable and that it could have any signal strength or length scale.

The state of knowledge about the constraint function $z$ is represented as the probability that the constraints are satisfied $p(z = 1|x, \theta)$. The simplest such model is the logistic regression:

$$p(z = 1|x, \lambda, b) = \text{sigm}(\lambda(x - b)) := \frac{1}{1 + e^{-\lambda(x - b)}},$$
with parameters $\lambda$ and $b$. As with the parameters of the covariance function, we do not assume that an individual knows the exact values of $\lambda$ and $b$. In other words, $\lambda$ and $b$ are not a part of the description of an individual’s type $\theta$. What is a part of the type $\theta$, however, is the individual’s prior beliefs about $\lambda$ and $b$. Specifically, we assume that an individual of type $\theta$ assigns a factorizing prior on $\lambda$ and $b$:

$$p(\lambda, b|\theta) = p(\lambda|\theta)p(b|\theta),$$  \hspace{1cm} (6)

and that each factor $\alpha \in \{\lambda, b\}$ is a Gaussian:

$$\alpha|\theta \sim N(\mu_\alpha, \sigma^2_\alpha),$$  \hspace{1cm} (7)

with given mean $\mu_\alpha$ and variance $\sigma^2_\alpha$. In other words, the specification of an individual’s type contains these parameters, i.e., $\{\mu_\lambda, \sigma^2_\lambda, \mu_b, \sigma^2_b\} \subset \theta$. While $\lambda$ quantifies the slope of the regression curve (refer to Equation 5), the parameter $b$ has an intuitive interpretation that correlates with the constraint knowledge of an individual. We discuss the interpretation of $b$ in Section 3.3.

### 2.2.4 Modeling how Individuals update their State of Knowledge

At step $t$, after an individual samples $x_t$, they receive information $(x_t, y_t, z_t)$. The individual processes this information by updating their belief about $f(x)$ and $g(x)$. However, due to their limited cognitive capabilities they may not be able to fully characterize all posteriors. The assumptions A1.2 and A2.2 imply that when they cannot deal with the computational complexity, they choose to obtain a maximum a posteriori estimate of their hyperparameters. Thus, an individual of type $\theta$ updates their beliefs about $f(x)$ to:

$$f(x)|\mathcal{I}_t, \theta \sim \text{GP} \left( m_t(x), c_t(x, x') \right)$$  \hspace{1cm} (8)

where $m_t(x)$ and $c_t(x, x')$ are the posterior mean and covariance functions of the GP [20] when it is conditioned on the $x$-$y$ input-output pairs contained in $\mathcal{I}_t$ that satisfy the constraints, i.e., on $\{(x_i, y_i) : (x_i, y_i, z_i) \in \mathcal{I}_t, z_i = 1, i = 0, \ldots, t\}$. Since we have assumed that the individuals have flat priors on the hyperparameters of the covariance function, $p(s, \ell|\theta) \propto 1$, this is equivalent to maximizing the marginal likelihood. Note that at the very first step, $t = 0$, the marginal likelihood
is flat with respect to the lengthscale. In that case, we assume that they pick \( \ell = 1 \). This may seem ad hoc, but it is inconsequential since their first decision at \( t = 1 \) does not depend on \( \ell \).

Similarly, the individuals use \( \mathcal{I}_t \) to update their state of knowledge about the feasible region, which is modeled as a logistic regression depending on \( b \) and \( \lambda \). In this part, their prior state of knowledge specified by their type \( \theta \), and specifically by \( \{\mu_\lambda, \sigma^2_\lambda, \mu_b, \sigma^2_b\} \subset \theta \), does play a role in their decision to choose the next search point. The modeling assumption is that they choose \( b \) and \( \lambda \) by maximizing the posterior of these hyperparameters, i.e., they choose:

\[
\hat{b}(\mathcal{I}_t; \theta), \hat{\lambda}(\mathcal{I}_t; \theta) = \arg \max_{b,\lambda} \prod_{i=0}^{t} p(z_i | x_i, \lambda, b)p(\lambda, b | \theta),
\]

(9)

### 2.2.5 Modeling how Individuals make Information Acquisition Decisions

To model how an individual of type \( \theta \) samples \( x_t \), we define a decision function \( \chi_t(\mathcal{I}_t; \theta) \). In the context of the SIADM scenario, the decision of what information to sample can be modeled utilizing a decision function. For example, for a convex search problem, a designer may choose to sample \( x_t \) based on a convex optimization method such as the bisection method. Then the decision function is modeled such that the next \( x \) is chosen at the mid point of the search space.

To descriptively formulate our decision function, our first assumption is that the decision function is not stationary i.e., it changes with \( t \). However, it only changes due to the information observed until step \( t \) i.e. \( \chi_t(\mathcal{I}_t; \theta) = \chi(\mathcal{I}_t; \theta) \). We do so to account for the argument that when \( t \) is small, individuals may wish to explore the space and that when \( t \) gets closer to \( T \), they wish to exploit their state of knowledge. The second assumption is that the decision function is myopic, i.e., it only considers the optimality of the next decision and not the optimality of the subsequent sequence of decisions. This assumption is reasonable, since individuals do not have the cognitive capabilities to think about many steps ahead [24]. There are many possible choices of myopic decision functions. Here, we opted for one of the most parsimonious models (no new parameters) which is based on the conditional expected improvement [25]. Borji and Itti [23] show that maximization of expected improvement is indicative of how humans make search decisions. It is:

\[
\chi(\mathcal{I}_t; \theta) = \arg \max_x \text{EI}(x; \mathcal{I}_t, \theta)p(z = 1 | x, \hat{\lambda}(\mathcal{I}_t; \theta), \hat{b}(\mathcal{I}_t; \theta)),
\]

(10)
where \( EI(x; \mathcal{I}_t, \theta) \) is the expected improvement (EI) defined via the GP representation of the objective function as modeled by an individual of type \( \theta \) and \( p(z = 1|x, \lambda(\mathcal{I}_t; \theta), b(\mathcal{I}_t; \theta)) \) is the probability that the constraints are satisfied given by the logistic regression function. \( \hat{\lambda}(\mathcal{I}_t; \theta) \) and \( \hat{b}(\mathcal{I}_t; \theta) \) are parameters that the individual has identified as described in Section 2.2.4.

### 2.3 A Researcher’s Belief about the Individual

A researcher is an individual who is observing the decision maker’s decisions but does not know their type \( \theta \). In Section 2.2, we formulate the beliefs of an individual about \( f(x) \) and \( g(x) \). In this section, we formulate the researcher’s beliefs about observing the decision data of an individual with type \( \theta \).

The researcher’s probability that an individual of type \( \theta \) selects \( x_t \) at the \( t \)-th step after having observed information \( \mathcal{I}_t \) is:

\[
p(x_t|\mathcal{I}_t, \theta) \sim \mathcal{N}(\chi(\mathcal{I}_t; \theta), \sigma^2),
\]

where \( \sigma^2 \in \theta \) is a type parameter that accounts for the deviation of an individual from the information acquisition strategy modeled in Section 2.2. Thus, for a fixed number of tries \( T \), the researcher’s probability of observing a sequence of decisions made by an individual \( x_{1:T} = \{x_1, \ldots, x_T\} \) is given by:

\[
p(x_{1:T}|y_{1:T}, z_{1:T}, \theta) = \prod_{t=0}^{T-1} p(x_{t+1}|\mathcal{I}_t, \theta)
\]

We refer to Equation 12 as the likelihood. Note that the likelihood is also conditioned on fixing the number of tries (assumption A3). Therefore, the probability of an individual for not stopping for \( T - 1 \) tries as well as the probability of stopping at \( T \) tries is 1.

The researcher’s prior beliefs about the type \( \theta \) of an individual are:

\[
p(\theta) = p(\mu_\lambda)p(\sigma_\lambda)p(\mu_b)p(\sigma_b)p(\sigma),
\]

where, the variances \( \sigma_\lambda^2 \), \( \sigma_b^2 \), and \( \sigma^2 \) are assigned an uninformative Jeffrey’s prior, e.g., \( p(\sigma_\lambda) \propto \frac{1}{\sigma_\lambda} \),

\[
\mu_\lambda \sim \mathcal{N}(-0.1, 0.001),
\]
Information at step $t$:

$$i = 1, \ldots, t$$

Figure 3: Graphical illustration of Sequential Information Acquisition and Decision Making (SIADM) model at step $t$. Parameters $\lambda, b, l, s$ are inferred by the individual. Parameters $\mu_b, \sigma_b, \mu_\lambda, \sigma_\lambda, \sigma$ are a part of an individual’s type $\theta$.

and

$$\mu_b \sim \mathcal{N}(300, 5000). \quad (15)$$

We assume that all the participant’s prior belief about $\mu_b$ and $\mu_\lambda$ is normally distributed as shown in Equation 14 and 15. These priors are assigned in accordance with the range of values chosen for the design of the experimental constraints as described in Section 3.1.1.

Figure 3 illustrates the information observed by the individual. Note that the researcher observes the entire information set $\mathcal{I}_t$. Figure 3 also illustrates the plate diagram of the SIADM model and the influence of various model parameters on the information acquired.
2.4 Inferring an Individual’s Type from Experimental Data

Our stochastic model of sequential decision making contains five parameters that specify the type of an individual, i.e., \( \theta = \{\mu_\lambda, \sigma^2_\lambda, \mu_b, \sigma^2_b, \sigma^2\} \). In this section, we discuss how observed decisions \( x_{1:T} \) of the individual can be used to infer their type \( \theta \). Section 2.3 describes a generative model of \( x_{1:T} \) conditioned on \( \theta \), which enables us to compute the likelihood \( p(x_{1:T}|y_{1:T}, z_{1:T}, \theta) \) and the researcher’s prior beliefs about an individual’s type \( \theta \). Using Bayes rule, the researcher’s posterior over \( \theta \) conditioned on \( x_{1:T} \) is:

\[
p(\theta|I_T) \propto p(x_{1:T}|y_{1:T}, z_{1:T}, \theta)p(\theta).
\]

We sample from the posterior using the Metropolis-Hasting algorithm sampling [26] from the PyMC [27] Python module. We run the MCMC chain for 10000 iterations with a burn-in period of 2000 samples that are discarded. Equation 16 is used to estimate the researcher’s posterior over \( \theta \) for an individual given their (individual’s) search data.

3 Experimental Study

We require a behavioral experiment with a SIADM task to obtain an individual’s search data. Such data enables us to estimate an individual’s type \( \theta \) from the model formulated in Section 2. In order to study the impact of domain knowledge and problem framing in a SIADM scenario, we need to formulate and test related hypotheses. Thus, in this section, we describe the experimental tasks, structure, and design. We then discuss the quantification and measurement of the factors investigated utilizing the experimental study and formulate the hypotheses.

3.1 Experimental Tasks

We assume that a designer receives information about the design objective \( f(x) \) and constraint \( g(x) \) with certainty i.e., the information sources are not noisy. We also assume that an individual’s domain knowledge does not affect their belief about the objective function. Thus, we assign a GP prior belief about the objective function for an individual in our model (refer to Section 2.2). To ensure consistency of the experiment with the model, the objective function \( f(x) \) is mathematically
unknown to the participants. However, we assume that the domain knowledge affects an individual’s belief about the constraints $g(x)$.

Our experimental study has three constrained optimization tasks. The first task is formulated as a domain-dependent track design problem where the feasible design space is not explicitly specified. For brevity, we call this task a Track-Design-Problem where Constraint-is-Not-Specified (TDP\textsubscript{CNS}). The second task is the track design problem where the constraint is specified (TDP\textsubscript{CS}). The third task is formulated as a domain-independent function optimization problem (FOP). The TDP\textsubscript{CNS} and FOP are mathematically identical but framed in different domain contexts. We do so to test the impact of problem framing on the participants’ decisions in SIADM scenarios. The TDP\textsubscript{CS} is formulated in order to understand the impact of adding a constraint. Table 1 illustrates the differences between each task. We do not consider a domain-independent task where the constraint is specified as it simply becomes a search task without influence of domain knowledge due to lack of a domain context and the constraint. In the following, we discuss the details of each design task.

Table 1: Differences between the Track Design Problem (TDP) and Function Optimization Problem (FOP)

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Domain</th>
<th>Dependent</th>
<th>Independent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specified</td>
<td>TDP\textsubscript{CS}</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>Not Specified</td>
<td>TDP\textsubscript{CNS}</td>
<td>FOP</td>
<td></td>
</tr>
</tbody>
</table>

3.1.1 Track Design Problems

The track design problems TDPs are formulated as SIADM tasks. The designer’s task is to design a roller coaster track where the objective $f(x)$ of the designer is to “maximize enjoyment experienced by the rider of the track” under the constraint $g(x)$ that the centripetal acceleration should not exceed $4g$. To achieve the objective, a participant’s task is to design a circular valley segment of the track with an appropriate width $w$. The participants are not provided an explicit mathematical form of the “enjoyment function” $E(w)$. However, they are informed that a small valley width would make the ride uncomfortable due to high $g$ forces and a wide valley has a high radius of curvature. Both cases result in reduced enjoyment. Thus, there is an optimal width $w$ for which the enjoyment for the rider is maximized. The participants are provided with an initial height $H$
of the track and are informed that the circular valley has a constant depth of 50 units as shown in Figure 4.

If the participants violate the constraint such that the centripetal acceleration exceed $4g$ then the track fails because the ride becomes uncomfortable due to high $g$ forces. In such a case, it is counter intuitive to display an “enjoyment” value. Thus, we chose a modeling scenario where violation of constraints results in no information about the objective function.

We design the objective function $E(w)$ such that it satisfies requirements such as concavity, non-negativity, function parameterization, and function asymmetry in order to control for factors such as incentivization, intuition, guessing, and problem difficulty to avoid interference with the experiment results. The details of the function characteristics are provided in the Appendix A. Considering such characteristics, we model enjoyment function through a Log-Normal function. The enjoyment ($E(w)$) of the track is defined as:

$$E(w) = 0.075 \exp(0.005 \frac{H^2}{w}) \exp \left\{ -\frac{(\ln (w) - \ln (H) - \ln (0.6) - 0.01)^2}{0.02} \right\}. \quad (17)$$

The maximum value of enjoyment function occurs at the width value $w_{max}$. We model $w_{max}$ as a function of the track height $H$ such that $w_{max} = 0.6H$. The corresponding maximum enjoyment value $E_{max}$ is:
\[ E_{\text{max}} = \left( \frac{H}{8} \right). \]  

(18)

The function is normalized to have a maximum value dependent on the height of the track. We do so because intuitively a “taller” ride should have a higher maximum possible enjoyment. In the experiment, the height values \( H \) are uniformly chosen from the range of 400 to 800 units. Thus, \( E_{\text{max}} \) values range between 50 to 100. However, the design alternative at \( w_{\text{max}} \) may still be infeasible i.e., not satisfy the acceleration constraint. The constraint is chosen by considering the standard safety measures adopted in general in a roller coaster track design where the \( g \) forces are limited between \(-4g\) and \(4g\). In the valley, the \( g \) force is always positive and therefore limited between 0 to \(4g\). The track is also assumed to be frictionless.

Mathematically, the problem can be formulated as an optimization problem as follows:

\[
\begin{align*}
\text{maximize} & \quad E(w) = 0.075 \exp (0.005) \frac{H^2}{w} \exp \left\{ - \left( \ln \left( \frac{w}{H} \right) - \ln \left( \frac{0.6}{0.01} \right) \right) \right\}, \\
\text{subject to} & \quad w^2 \geq 200H.
\end{align*}
\]  

(19)

Participants were not aware of the explicit form of the objective function \( E(w) \) as seen in Equation 17. An understanding of laws of motion, centripetal acceleration and force balance is required to formulate the constraint in Equation 19. We assume that an individual with knowledge of the Newtonian concept of force will be able to formulate the constraint in Equation 19. This assumption is reasonable as research has shown that individuals with high domain knowledge tend to categorize a problem according to the major concept that could be applied to solve the problem[28].

Consider a randomly chosen \( H \) value of 500. The theoretical maximum value of the function is \( E_{\text{max}} = 62.5 \) at the width value of \( w_{\text{max}} = 0.6H = 300 \). However, for the constraint to be satisfied we need \( w \geq 316.23 \). Thus, the function maximum is not a feasible solution and the optimal lies at the constraint boundary in this case. Such cases are included in order to reduce learning [29] about the relationship between \( w_{\text{max}} \) and track parameter \( H \). For track design problem where constraint is specified \( TDP_{CS} \) the participants were additionally given the constraint information. We do so by giving the range of width values for which the solution would be feasible. Figure 4 illustrates \( TDP_{CS} \).
3.1.2 Function Optimization Problem

We design the function optimization problem by excluding the context of the track design task. In the function optimization problem, the participants are asked to maximize a concave function \( f(x) \) given a constraint function \( g(x) \). Their task is to sample values to obtain the maximum value of \( f(x) \) as well as ensure that \( g(x) < 2 \) for a feasible solution (choosing a set of design variables \( x \) is referred to as sampling). The objective function \( f(x) \) remains exactly the same as the objective function of the track design game, \( E(x) \) (Equation 17). The constraint in the TDP is such that the centripetal acceleration is less than 4\( g \) (and greater than 0). The centripetal acceleration constraint is shown as a constraint function \( g(x) \) in FOP and the values are normalized between 0 and 2 to minimize learning about the mathematical similarity of the tasks.

3.2 Experiment Design

The experiment involved a total of 44 participants. These participants were undergraduate and graduate students at a large public university. The participants were engineering majors from various departments such as mechanical, civil, chemical, and nuclear engineering. The experiment was divided into four parts. In the first part, the participants were required to take a Concept Inventory test (the details are provided in Section 3.3). No time limit was imposed for this part. In the rest of the parts, each participant was required to play the games in a predetermined order. There were a total of three orders of execution of the experiment tasks. Such ordering of tasks is done to eliminate order effects [30]. There can be six possible permutations of the order of execution of three experimental tasks. However, we eliminated certain cases as follows: In any task we did not want the participants to play TDP\(_{CS} \) before playing TDP\(_{CNS} \). This was done to eliminate learning effects [29]. As constraints are explicitly specified in TDP\(_{CS} \), knowledge of the constraints may interfere with the performance in TDP\(_{CNS} \). Thus, we eliminate three of the permutations of the order of execution of the three experiment tasks.

Table 2: Treatments and number of participants in each treatment

<table>
<thead>
<tr>
<th>Treatment Order</th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDP(<em>{CNS} ) - FOP(</em>{CNS} ) - TDP(_{CS} )</td>
<td>15 participants</td>
</tr>
<tr>
<td>TDP(<em>{CNS} ) - TDP(</em>{CS} ) - FOP(_{CNS} )</td>
<td>15 participants</td>
</tr>
<tr>
<td>FOP(<em>{CNS} ) - TDP(</em>{CNS} ) - TDP(_{CS} )</td>
<td>14 participants</td>
</tr>
</tbody>
</table>

Panchal 19 MD-18-1187
Each order of the experimental tasks, as shown in Table 2, is termed as a *treatment*. Each participant was a part of one of these three treatments. Each experimental task within a treatment consisted of 7 *periods*. In each period, the objective function was randomly generated. In particular, the objective function parameter \( H \) was randomly chosen from a uniform distribution between 400 to 800 units.

Each period consisted of seven (7) fixed number of tries. A *try* is defined to be a submission of one sampled \( w \) (or \( x \) for FOP) value. A *successful try* is defined as one in which the constraint in Equation 19 is satisfied. Otherwise, the try is termed unsuccessful. For all the tasks, at the end of each successful try, the value of the objective function was shown. Additionally, for TDP, an animation of the ride was shown to the participants.

The incentive structure for the participants was designed as follows. Participants were paid based on their performance to align incentives with the task objective of obtaining the maximum value. We do so by obtaining a ratio of the maximum function value obtained by the participant in a period to the actual maximum achievable value. This ratio is multiplied with a constant value of $2.5. Thus, the participants can achieve a maximum incentive of $2.5 in each period. To control for wealth effects [31], the participants are informed that for any task they would be paid for their performance in two randomly chosen periods of that task. As there are a total of three tasks the participants can earn a maximum of $15. Additionally, they are given $5 as a participation fee.

### 3.3 Metrics Utilized for Hypothesis Formulation and Testing

We describe the metrics utilized to quantify the factors under investigation in this study. We list these factors in Table 3.

We consider domain knowledge as the general conceptual knowledge of an individual about a specific domain. We quantify an individual’s domain knowledge through the test-scores of a Concept Inventory. In this specific study, we utilize the test-scores \( S \) of the Force Concept Inventory (FCI) [17]. The FCI quantifies an individual’s knowledge of Newtonian concepts of force. These concepts are required to comprehend the constraints in the track design tasks. The FCI has been validated utilizing Item Response Theory [32]. An FCI score ranges from 0 to 30. A score of less than 15 is considered as a low score [32]. Shergadwala et al. [33] discuss how FCI scores can be utilized to assess the performance of individuals in a design context. Other metrics such as
Table 3: List of factors under investigation in this study and their method of measurement

<table>
<thead>
<tr>
<th>Factor</th>
<th>Method of Measurement or Control</th>
<th>Measure or Control</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domain Knowledge</td>
<td>Concept Inventory Scores</td>
<td>$S$</td>
<td>Performance is measured by averaging all the function values sampled in a given period over the total number of tries in the period</td>
</tr>
<tr>
<td>Lack of Knowledge</td>
<td>Experimental Data and Model</td>
<td>$\mu_{b_{\text{diff}}}$</td>
<td></td>
</tr>
<tr>
<td>Deviation from ideal strategy</td>
<td>Experimental Data and Model</td>
<td>$\sigma$</td>
<td></td>
</tr>
<tr>
<td>Constraint Specification</td>
<td>Constraint is either specified or not in the problem statement</td>
<td>Experimental Control</td>
<td></td>
</tr>
<tr>
<td>Problem Framing</td>
<td>Problem is framed as a Track Design Task and a Function Maximization Game</td>
<td>Experimental Control</td>
<td></td>
</tr>
</tbody>
</table>

student’s GPA or subject specific grades cannot be utilized to assess their performance as they are inconsistent across universities and lack verification.

An individual’s lack of knowledge ($\mu_{b_{\text{diff}}}$) is defined as the distance of an individual’s belief about the location of the constraint boundary from the actual location. The hyperparameter $\mu_b$ represents an individual’s mean prior belief about the location of the constraint boundary. The actual location of the constraint boundary is $b_{\text{actual}} = \sqrt{200H}$ (refer to Equation 19). Thus, an individual’s lack of knowledge is quantified as,

$$
\mu_{b_{\text{diff}}} = |\mu_b - b_{\text{actual}}|.
$$

As $\mu_{b_{\text{diff}}}$ estimates the distance between actual constraint boundary $b_{\text{actual}}$ and the hyperparameter $\mu_b$, intuitively a smaller $\mu_{b_{\text{diff}}}$ means a lesser lack of knowledge. This implies a better state of knowledge about the constraints. It is to be noted that $\mu_{b_{\text{diff}}}$ is specific to the class of problems where there are inequality constraints.

We quantify an individual’s deviation from the modeled SIADM strategy through the hyperparameter $\sigma$. The decision-making data of the individuals obtained from the experiments is utilized to infer the parameters $\mu_b$ and $\sigma$ as discussed in Section 2.4. Problem framing is controlled by formulating the Track Design Problem TDP and Function Optimization Problem FOP.

The performance of an individual is measured as follows: For a given objective function in a given period we average the $f(x)$ values over all the tries (tries and periods are defined in
Section 3.2). For example, if an individual sampled \( \{x_1, \ldots, x_T\} \) sequentially in a design space to receive \( \{f(x_1), \ldots, f(x_T)\} \) then the average of the \( f(x) \) values achieved over \( T \) tries is considered as the person’s performance. We do so to reduce the effects of guessing the maximum value or randomly sampling a high function value.

### 3.4 Hypotheses Formulation and Operationalization

We list all the hypotheses and their corresponding operationalization in Table 4.

**Table 4: Operationalization of Hypotheses**

<table>
<thead>
<tr>
<th>Research Objective</th>
<th>Hypotheses</th>
<th>Operationalized Hypotheses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 To quantify the impact of domain knowledge on SIADM process and outcomes.</td>
<td>H1: Domain knowledge affects the initial state of knowledge of a SIADM task.</td>
<td>H1*: Average ( \mu_{\text{diff}} ) is a decreasing function of the FCI score.</td>
</tr>
<tr>
<td></td>
<td>H2: Domain knowledge affects design performance.</td>
<td>H2*: Average enjoyment value achieved by a participant in the Track Design Game is an increasing function of their FCI score.</td>
</tr>
<tr>
<td></td>
<td>H3: Domain knowledge affects SIADM strategy.</td>
<td>H3*: Average ( \sigma ) value is a decreasing function of the FCI score.</td>
</tr>
<tr>
<td>1.2 To quantify the impact of problem framing on the SIADM outcomes.</td>
<td>H4: Participants will have a better state of knowledge about a domain dependent problem as compared to a domain-independent problem.</td>
<td>H4*: Participants will have a lower average ( \mu_{\text{diff}} ) in the Track Design Game than in the Function Maximization game.</td>
</tr>
<tr>
<td></td>
<td>H5: Participants will have a better performance in a domain dependent problem as compared to a domain-independent problem.</td>
<td>H5*: Participants will have a higher average function value in the Track Design Game than in the Function Maximization game.</td>
</tr>
<tr>
<td></td>
<td>H6: Participants will have a better state of knowledge about the problem where constraints are specified as compared to a problem where constraints are not specified.</td>
<td>H6*: Participants will have lower average ( \mu_{\text{diff}} ) in the Track Design Game where the information about the constraint is specified (TDP\text{CS}) as compared to the Track Design Game where the information about the constraint is not specified (TDP\text{CNS}).</td>
</tr>
<tr>
<td></td>
<td>H7: Participants will have a better performance in a problem where constraints are specified as compared to a problem where constraints are not specified.</td>
<td>H7*: Participants will have a higher average enjoyment value in the Track Design Game where the information about the constraint is specified (TDP\text{CS}) as compared to the Track Design Game where the information about the constraint is not specified (TDP\text{CNS}).</td>
</tr>
<tr>
<td></td>
<td>H8: Problem framing impacts SIADM strategy.</td>
<td>H8*: Participants have a lower average ( \sigma ) in the Track Design Game than the Function Maximization game.</td>
</tr>
</tbody>
</table>

We recall the discussion in Section 2.1 and reiterate that the state of knowledge of an individual at each step \( t \) in a SIADM process affects the decisions in the next step \( (t + 1) \). Therefore, we
consider H1* and H2* where conditional on the design task to encompass the domain knowledge (Newtonian force concept in this case) we hypothesize that the FCI score will negatively correlate with the lack of knowledge parameter $\mu_b^{\text{diff}}$ and positively correlate with performance.

The “decision making error” $\sigma$ can be considered as the deviation of an individual’s search strategy from the assumed search strategy. We hypothesize (H3*) that an individual with a higher domain knowledge will closely follow the modeled strategy.

By framing the same mathematical problem as a track design task and a function maximization task we hypothesize (H4*) that the participants will have a better understanding about the track design task which implies a smaller $\mu_b^{\text{diff}}$ for the track design task. As a consequence of better understanding the track design task, we also hypothesize (H5*) that the participants will have a better performance in the track design task as compared to the function optimization problem.

As $\mu_b^{\text{diff}}$ quantifies the belief about constraints, we formulate H6*. Since the participants are given the constraint boundary in TDP$_{CS}$ the $\mu_b^{\text{diff}}$ will be smaller as compared to TDP$_{CNS}$. As the information about the constraint boundary is provided, participants will be able to make better decisions and perform better. Thus, we formulate and operationalize H7*. We formulate H8* by hypothesizing that participants will follow the modeled strategy closely for a domain-specific task as compared to a domain-independent task.

4 Results and Discussion

We utilize the data, collected from the experiment described in Section 3, to infer the model parameters $\theta$. Based on these parameters and the experimental data, we test hypotheses H1* to H8* in this section. We then discuss the implications of each of the hypothesis test results. Table 4 categorizes H1* to H8* with respect to our research objective that is divided into two parts. The first part of the research objective is related to the domain knowledge and the second part is concerned with problem framing.

4.1 Hypotheses Testing

We categorize H1* through H3* as hypotheses related to person-specific factors that are conceptual knowledge (FCI Score) and lack of knowledge about the constraints ($\mu_b^{\text{diff}}$). H4* to H8* are cate-
gorized as hypotheses related to problem-specific factors such as problem framing and constraint specifications in the problem. For each individual, their $\mu_b^{\text{diff}}, \sigma^2$, and performance are averaged over all but first two periods. As the participant gets familiar with the task we disregard data from the first two periods for each task to reduce the impact of learning effects [29].

4.1.1 Hypotheses Testing: Person-specific factors

To understand the impact of person-specific factors (FCI score and $\mu_b^{\text{diff}}$) in a SIADM task, we test $H1^*$ through $H3^*$. We investigate the impact of knowledge about the Newtonian concepts of force (quantified by the FCI scores) only in the tasks that require that conceptual knowledge. As FOP does not require knowledge of Force Concepts we do not investigate the impact of FCI scores on an individual’s performance in FOP.

We test $H1^*$ by conducting a regression analysis between FCI scores and $\mu_b^{\text{diff}}$. The results of $H1^*$ indicate that there is no significant linear relationship between FCI score and the lack of knowledge about the constraints $\mu_b^{\text{diff}}$ ($p = 0.22 > 0.05$). The ANOVA results for FCI score versus $\mu_b^{\text{diff}}$ is shown in Table 5. The scatter plots for FCI score versus $\mu_b^{\text{diff}}$ are shown in Figure 5.

Table 5: ANOVA of FCI score and $\mu_b^{\text{diff}}$ in track design tasks.

<table>
<thead>
<tr>
<th>Metric</th>
<th>ANOVA with $\mu_b^{\text{diff}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCI Score</td>
<td>TDP$_{\text{CNS}}$</td>
</tr>
<tr>
<td>$S$</td>
<td>$r = 0.19$</td>
</tr>
<tr>
<td></td>
<td>$p = 0.22$</td>
</tr>
<tr>
<td></td>
<td>TDP$_{\text{CS}}$</td>
</tr>
<tr>
<td></td>
<td>$r = -0.24$</td>
</tr>
<tr>
<td></td>
<td>$p = 0.11$</td>
</tr>
</tbody>
</table>

Table 5 illustrates that the participant’s lack of knowledge about the constraints ($\mu_b^{\text{diff}}$) was not
significantly affected by their conceptual knowledge of Newtonian force concepts (FCI scores). We recognize that FCI scores and $\mu_b^{diff}$ are direct and indirect measures of an individual’s knowledge about the problem, respectively. The impact of domain knowledge on the state of knowledge may be reduced due to learning effect [29].

We test $H_2^*$ by conducting a regression analysis between FCI scores and performance. The ANOVA results for FCI score versus performance are shown in Table 6. The scatter plots for FCI score versus performance are shown in Figure 6. The figure illustrates that individuals with a greater FCI score have a higher performance with lesser variation. This implies that individuals with a higher FCI score are likely to perform better. In both the track design tasks, the results of $H_2^*$ indicate a significant ($p < 0.05$) weak positive linear relationship ($0.25 < r \leq 0.5$) between FCI scores and performance.

Table 6: ANOVA of FCI score and performance in track design tasks.

<table>
<thead>
<tr>
<th>Metric</th>
<th>ANOVA with performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>FCI Score</td>
<td>TDP$_{CNS}$</td>
</tr>
<tr>
<td>$S$</td>
<td>$r = 0.31$</td>
</tr>
<tr>
<td>$p = 0.042$</td>
<td>$p &lt; 0.001$</td>
</tr>
</tbody>
</table>

Figure 6: Scatter plots for $H_2^*$

Table 6 indicates that there is a significant ($p < 0.05$) linear relationship between an individual’s conceptual knowledge of the Newtonian Concepts of Force and the TDP tasks that require such knowledge. However, as the correlation is weak ($0.25 < r \leq 0.5$) the FCI scores cannot be solely utilized to predict an individual’s performance in different SIADM scenarios. While FCI score may be a good metric for conceptual knowledge quantification, further investigation is required to
differentiate direct and indirect measures of knowledge and their subsequent impact on SIADM outcomes.

We test H3* by conducting a regression analysis between FCI scores and $\sigma$. The results of H3* indicate that there is a significant ($p < 0.05$) linear relationship between FCI score and the deviation ($\sigma$) of an individual from the modeled strategy for TDP$_{CS}$. However, as the correlation is weak ($0.25 < r \leq 0.5$) the FCI scores cannot be solely utilized to predict an individual’s deviation from the SIADM model. The p-value for TDP$_{CNS}$ is less than the level of significance $\alpha = 0.05$. The ANOVA results for FCI score versus $\sigma$ are shown in Table 7. The scatter plots for FCI score versus $\sigma$ are shown in Figure 7.

Table 7: ANOVA of FCI score and $\sigma$ in track design tasks.

<table>
<thead>
<tr>
<th>Metric</th>
<th>ANOVA with $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDP$_{CNS}$</td>
<td>$r = -0.26$</td>
</tr>
<tr>
<td>TDP$_{CS}$</td>
<td>$r = -0.38$</td>
</tr>
</tbody>
</table>

![Figure 7: Scatter plots for H3*](https://example.com/figure7.png)

Table 7 illustrates that when the constraints are specified, an individual’s domain knowledge affects how closely the individual followed the modeled strategy. The variation of performance of the individuals with low FCI score is indicated in Figure 6. This variation may be explained by the results of H3* that people with higher FCI score tend to follow the modeled strategy closely, and that may impact their performance. A greater and significant correlation in the track design task where constraints are specified may be due to the order of the track design tasks. As TDG$_{CS}$ is always played after TDG$_{CNS}$, it may result in learning effect [29] such that the participants get
closer to the modeled strategy after repeatedly playing the track design task.

### 4.1.2 Hypotheses Testing: Problem-specific factors

To test H4* we compare the average $\mu_b^{\text{diff}}$ of an individual in TDP and FOP by conducting a paired two sample t-test. The hypothesis test results for H4 indicate that $\mu_b^{\text{diff}}$ is indeed lower in the Track Design Games than the Function Maximization Game ($p < 0.05$). Therefore, the state of knowledge about the constraints in the Track Design Game is better than the Function Maximization Game.

We conclude that problem framing impacts the lack of knowledge about the constraints. The results are shown in Table 9. The mean and variance of the average $\mu_b^{\text{diff}}$ value for TDP ($\alpha_{\mu}^{\text{TDP}}, \gamma_{\mu}^{\text{TDP}}$) and the mean and variance of the average $\mu_b^{\text{diff}}$ value for FOP$_{\text{CNS}}$ ($\alpha_{\mu}^{\text{FOP}}, \gamma_{\mu}^{\text{FOP}}$) are shown in Table 8.

**Table 8: Mean ($\alpha$) and variance ($\gamma$) of the average $\mu_b^{\text{diff}}$ values in TDP and FOP**

<table>
<thead>
<tr>
<th>Game</th>
<th>Average $\mu_b^{\text{diff}}$</th>
<th>Mean $\alpha$</th>
<th>Variance $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDP$_{\text{CNS}}$</td>
<td>$\mu_{\mu}^{\text{TDP}} = 59.59, \gamma_{\mu}^{\text{TDP}} = 499.46$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample size=44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TDP$_{\text{CS}}$</td>
<td>$\mu_{\mu}^{\text{TDP}} = 61.95, \gamma_{\mu}^{\text{TDP}} = 353.51$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample size=44</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FOP$_{\text{CNS}}$</td>
<td>$\mu_{\mu}^{\text{FOP}} = 79.11, \gamma_{\mu}^{\text{FOP}} = 1694.84$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample size=44</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 9: Summary of the two sample t-test for H4***

<table>
<thead>
<tr>
<th>Alternate Hypothesis</th>
<th>t stat.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two sample t-test for TDP$_{\text{CNS}}$</td>
<td>$\alpha_{\mu}^{\text{FOP}} &gt; \alpha_{\mu}^{\text{TDP}}$</td>
<td>-2.66</td>
</tr>
<tr>
<td>Two sample t-test for TDP$_{\text{CS}}$</td>
<td>$\alpha_{\mu}^{\text{FOP}} &gt; \alpha_{\mu}^{\text{TDP}}$</td>
<td>-2.33</td>
</tr>
</tbody>
</table>

We failed to reject the null for H6* ($p = 0.6$) and H7* ($p = 0.59$). This means that while $\mu_b^{\text{diff}}$ is indicative of an individual’s lack of knowledge about the constraints in domain dependent and independent task, it does not capture the effect of providing information about the constraints for the same task. Learning about the task may interfere with providing information about constraints in one task and not in the other. If the lack of knowledge about the constraints is descriptively captured through $\mu_b^{\text{diff}}$, its value should ideally be zero when the participants know exactly where the constraint boundary lies. However, we do not observe a significant difference between $\mu_b^{\text{diff}}$ values in TDP$_{\text{CNS}}$ and TDP$_{\text{CS}}$.

To test $H5^*$ we compare the performance of the participants in both the track design games with
the function maximization game. We conducted a paired two sample t-test. The results are shown in Table 10. Both the p-values are less than the level of significance (\(\alpha = 0.05\)). This indicates that the performance of the participants was indeed better in both the track design tasks as compared to the function maximization task. The mean and variance of the average enjoyment value for TDP \((\alpha_E, \gamma_E)\) and the mean and variance of the average function value for FOP_{CNS} \((\alpha_F, \gamma_F)\) are shown in Table 11.

Table 10: Summary of the two sample t-test for \(H^5\)

<table>
<thead>
<tr>
<th>Alternate Hypothesis</th>
<th>t stat.</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two sample t-test for TDP_{CNS}</td>
<td>(\alpha_E &gt; \alpha_F)</td>
<td>12.10</td>
</tr>
<tr>
<td>Two sample t-test for TDP_{CS}</td>
<td>(\alpha_E &gt; \alpha_F)</td>
<td>5.90</td>
</tr>
</tbody>
</table>

Table 11: Mean \((\alpha)\) and variance \((\gamma)\) of the average enjoyment values \(E\) in TDP and average function values \(F\) in FOP

<table>
<thead>
<tr>
<th>Game</th>
<th>Average output value</th>
<th>Mean (\alpha)</th>
<th>Variance (\gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDP_{CNS}</td>
<td></td>
<td>(\alpha_E = 88.54)</td>
<td>(\gamma_E = 141.75)</td>
</tr>
<tr>
<td>Sample size=44</td>
<td></td>
<td>(\alpha_E = 75.36)</td>
<td>(\gamma_E = 428.34)</td>
</tr>
<tr>
<td>TDP_{CS}</td>
<td></td>
<td>(\alpha_E = 58.61)</td>
<td>(\gamma_E = 369.43)</td>
</tr>
<tr>
<td>Sample size=44</td>
<td></td>
<td>(\alpha_E = 58.61)</td>
<td>(\gamma_E = 369.43)</td>
</tr>
<tr>
<td>FOP_{CNS}</td>
<td></td>
<td>(\alpha_F = 58.61)</td>
<td>(\gamma_F = 369.43)</td>
</tr>
<tr>
<td>Sample size=44</td>
<td></td>
<td>(\alpha_F = 58.61)</td>
<td>(\gamma_F = 369.43)</td>
</tr>
</tbody>
</table>

The results shown in Table 10 indicate that problem framing of the same mathematical task does indeed affect the task performance [16]. We do not find a significant difference between performance in the two track design tasks. The result indicates that providing information about the design space did not significantly impact the task outcome. The result could be influenced due to lack of control over search strategies of different individuals in the track design tasks as well as the learning effect [29]. There could also be a potential for bias in the second track design game play (TDP_{CS}) due to the previous track design game play (TDP_{CNS}).

We failed to reject the null of H8* \((p = 0.23)\). Thus, there is no significant difference of the average \(\sigma\) in the Track Design Games and the Function Maximization Game. We cannot conclude that problem framing impacts an individual’s deviation from the modeled SIADM strategy.
Table 12: Summary of Hypothesis Results. ✓ indicates rejection of null and ✗ indicates failure of null rejection.

<table>
<thead>
<tr>
<th>Hypotheses</th>
<th>Results</th>
<th>Hypotheses</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>(H1*)</td>
<td>For TDP&lt;sub&gt;CS&lt;/sub&gt; : ✗ For TDP&lt;sub&gt;CNS&lt;/sub&gt; : ✓</td>
<td>(H5*)</td>
<td>✓</td>
</tr>
<tr>
<td>(H2*)</td>
<td>For TDP&lt;sub&gt;CS&lt;/sub&gt; : ✓ For TDP&lt;sub&gt;CNS&lt;/sub&gt; : ✓</td>
<td>(H6*)</td>
<td>✗</td>
</tr>
<tr>
<td>(H3*)</td>
<td>For TDP&lt;sub&gt;CS&lt;/sub&gt; : ✓ For TDP&lt;sub&gt;CNS&lt;/sub&gt; : ✓</td>
<td>(H7*)</td>
<td>✗</td>
</tr>
<tr>
<td>(H4*)</td>
<td>✓</td>
<td>(H8*)</td>
<td>✓</td>
</tr>
</tbody>
</table>

4.2 Hypotheses Tests: Discussion

We summarize the results of the hypothesis test in Table 12. The hypothesis test results indicate that framing a SIADM task in a domain specific context decreases an individual’s lack of knowledge about the problem constraints. Such problem framing also positively impacts an individual’s performance. We conclude that the FCI scores a weak but a significant correlation with an individual’s performance in a SIADM task and therefore can be utilized as preliminary indicator of the impact of domain knowledge on performance. Also, we do not find any significant differences in participant’s performance across treatments.

It is to be noted that the participants take the FCI test before solving the design problem. This may result in the participants inferring that the design problems involve Newtonian concepts of force. Such inference minimizes the impact of a participant’s ability to recall the force concepts and thus, it strengthens the validity of the results of the correlations between FCI scores and performance. The weak correlations indicate the need towards developing descriptive measures of knowledge such as $\mu^\text{diff}_b$.

From Hypothesis 4, we wanted to leverage the truth that participants will have a better state of knowledge about the domain-specific track design problem than about the function optimization problem due to problem framing. We indeed find that $\mu^\text{diff}_b$ is lower in the track design game than the function optimization game. Based on this, we verify that $\mu^\text{diff}_b$ is indicative of an individual’s state of knowledge. This result justifies the most important modeling assumption in this study that the distance of an individual’s belief about the location of the constraint boundary from the actual location represents their lack of knowledge about the task constraints.

We do not observe a relationship between conceptual knowledge and an individual’s lack of knowledge.
knowledge about the problem constraints. This observation may be because the FCI score of an individual remains the same throughout the study whereas $\mu_{b}^{\text{diff}}$ varies over successive periods. The variation of $\mu_{b}^{\text{diff}}$ implies variation of the lack of knowledge about the feasible design space. This variation may be due to the randomization of feasible design space and its impact on learning about the task problem over multiple periods of the game play.

5 Discussion

In this section, we discuss the SIADM framework as a contribution of this study. Then, we discuss about the validity of the research results and the generalizability of the model for extending the proposed framework.

5.1 Contributions

The primary contribution of this study is a SIADM framework which is instantiated by presenting a SIADM model in conjunction with a behavioral experiment for a class of design problems. Such a framework enables us to understand how individuals sequentially acquire information and make decisions in a design context. We quantify the impact of factors, such as problem framing and an individual’s lack of domain knowledge, on the SIADM outcomes of a design search problem with constraints. We find that problem framing impacts an individual’s knowledge about the problem constraints as well as their performance.

We represent a SIADM process as one that consists of three activities as illustrated in Figure 2 and described in Section 2.1. We make specific modeling choices for these three activities in our SIADM model as discussed in Section 2.2. Specifically, we assume that individuals maximize the expected improvement in the objective function, and follow a myopic one-step look-ahead strategy for calculating the expected improvement. Based on these assumptions, we study the impact of factors, such as problem framing and an individual’s lack of domain knowledge, on the SIADM outcomes.

The proposed model can be utilized to investigate behavioral similarities and differences among individuals. Specifically, individuals can be categorized based on the combinations of $\mu_{b}^{\text{diff}}$ and $\sigma$. In the future, their behavior and SIADM outcomes can be compared across such categorizes to
study the influence of both domain knowledge and following a particular SIADM model.

5.2 Validity

Our experimental study has high internal validity as it is a controlled behavioral experiment [34]. Internal validity refers to ensuring that the observed effect on the SIADM activities is attributable to the factors identified as a cause. We control for other factors such as an individual’s learning, intuition, the order of experiment task execution, incentivization of the experiment tasks, and the similarity of the search tasks in TDP and FOP that also affect a participant’s decision-making (refer to Section 3).

External validity refers to the generalization of the research study [35]. As in any controlled experiment, the external validity depends on how well the experimental conditions represent the target setting. The SIADM framework that consists of the three activities of a SIADM process, as illustrated in Figure 2, is highly general. Any sequential information acquisition activity in the design process can be represented using this framework. The SIADM model, on the other hand, is more specific because it has been instantiated for a particular class of design problems. These problems have a single objective and a single constraint with a single design variable. Consequently, the defined model parameter $\mu_b^{\text{diff}}$ is specific to the problems with single inequality constraints. In order to utilize the model in more complex design scenarios, various aspects of the proposed model such as its parameters and the SIADM activities will have to be appropriately considered. For example, in a design problem with multiple constraints, $\mu_b^{\text{diff}}$ could be considered as a set of parameters. Similarly, a problem with multiple objectives will impact the way an individual updates their beliefs about the objectives using Bayesian updating. Further investigation is required to evaluate the effects of complexity on the SIADM model formulation. We also acknowledge that in reality, individuals may cognitively execute the three activities in a SIADM scenario differently. We do not test whether the proposed model is representative of how individuals follow a SIADM process. To investigate the representativeness of an individual’s SIADM process by the proposed model there lies a need to develop alternate descriptive models of SIADM. Then, such models can be compared using Bayesian model comparison to evaluate which model best represents the decision making strategy followed by the individuals. This is a promising avenue for further research in this direction.
The external validity of the proposed framework can also be assessed by how well the model applies to different experimental settings such as (a) different populations (b) different design problems, and (c) different SIADM factors. Our experimental study has been carried out with undergraduate and graduate engineering students. It is not clear how well these results will extend to practicing engineers who have other implicit as well as procedural knowledge. In real life settings, SIADM scenarios are more complex with multiple objectives and multiple constraints. Our study does not account for the effects of complexity as a factor on SIADM scenarios. We do believe that it is likely that different ways of increasing complexity affects behaviors in different ways. As the complexity grows, other factors such as the manner in which information is presented also affects the behaviors. For example, if there are two or more variables, the visual representation of the acquired data ($x$ and $f(x)$) affects how individuals process information and make decisions. With increasing complexity, computational tools (e.g., surrogate models) are needed to support designers. The behavior then depends on the types of computational tools used. To assess the ecological validity in such settings, we can not only perform experiments but also conduct interviews, surveys, and case studies. All these effects cannot be captured in a single experiment. Therefore, the complexity of the problem and its effects on information acquisition strategies adopted by humans requires further investigation.

While our framework is developed for individual SIADM scenarios, it can be used as a component within more complex design settings such as in teams where multiple designers make decisions in parallel. For example, our framework can be used to model a team member making sequential decisions within a team. However, further investigation would be required to understand how the three activities of a SIADM process would be affected based on the interactions of an individual with their team members on every iteration step of a team member’s SIADM process.

**Acknowledgments**

The authors gratefully acknowledge financial support from the National Science Foundation through NSF CMMI Grants 1400050 and 1662230. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author and do not necessarily reflect the views of the National Science Foundation.
References


A Required Characteristics of the Objective Function

We require the enjoyment function to have the following characteristics:

1. The objective function should be concave so that there is a unique maximum value of enjoyment.

2. The objective function should be non-negative to ensure that the enjoyment value is non-negative.

3. The objective function should be sensitive to the values in the design space sampled by the user.

4. The concavity of the function should be such that it is less intuitive for the participant to achieve the maximum by sampling values at random. In this way the participants would also have an incentive in trying to converge to the optimum.

5. The enjoyment value should eventually decrease to zero by moving away from the optimal value.

6. The function should have flexible parameters to adjust its maximum value either on the constraint boundary or somewhere within the design space. This would make it less intuitive for the participant to search for the optimal by guessing.
7. The enjoyment function could be asymmetric with respect to the optimal width value. The rate at which enjoyment decreases due to increase of width could be different from the rate at which enjoyment decreases due to decreasing width.
List of Figures

1 Illustration of past research emphasis in Decision-based Design and the focus of this paper. ................................................................. 4
2 Illustration of Sequential Information Acquisition and Decision Making process. Decisions are highlighted in gray color. Rectangular nodes are information acquisition decisions and the outcome (diamond node) of the SIADM process is making the artifact decision. ................................................................. 7
3 Graphical illustration of Sequential Information Acquisition and Decision Making (SIADM) model at step $t$. Parameters $\lambda$, $b$, $l$, and $s$ are inferred by the individual. Parameters $\mu_b$, $\sigma_b$, $\mu_\lambda$, $\sigma_\lambda$, and $\sigma$ are a part of an individual’s type $\theta$. ................................. 14
4 The user interface of track design game where constraint is specified TDP$_{CS}$ .... 17
5 Scatter plots for H1*. ........................................................................... 24
6 Scatter plots for H2* ........................................................................... 25
7 Scatter plots for H3* ........................................................................... 26

List of Tables

1 Differences between the Track Design Problem (TDP) and Function Optimization Problem (FOP) ......................................................... 16
2 Treatments and number of participants in each treatment ......................... 19
3 List of factors under investigation in this study and their method of measurement . 21
4 Operationalization of Hypotheses .................................................................. 22
5 ANOVA of FCI score and $\mu_b^{\text{diff}}$ in track design tasks. ......................... 24
6 ANOVA of FCI score and performance in track design tasks. ..................... 25
7 ANOVA of FCI score and $\sigma$ in track design tasks. .................................. 26
8 Mean ($\alpha$) and variance ($\gamma$) of the average $\mu_b^{\text{diff}}$ values in TDP and FOP ......................................................... 27
9 Summary of the two sample t-test for H4* .................................................. 27
10 Summary of the two sample t-test for $H5^*$ .............................................. 28
11 Mean ($\alpha$) and variance ($\gamma$) of the average enjoyment values $E$ in TDP and average function values $F$ in FOP ......................................................... 28
Summary of Hypothesis Results. ✓ indicates rejection of null and ✗ indicates failure of null rejection.