ABSTRACT

Design of transmission gears is carried out, predominantly using the AGMA (American Gear Manufacturing Association) standards for gear design. The AGMA based gear design process includes a large number of empirical “design factors” accounting for various uncertainties related to geometry, precision requirements, material, manufacturing processes, mounting, quality requirements, reliability, etc. Many of these factors are based on large numbers of tests and many years of experience. As the knowledge of materials and service load conditions increases, it becomes possible to include the uncertainty in load and material properties systematically and reduce the empirical design factors used in AGMA gear design procedure. In this paper we propose a method for robust design and eliminate the factor of safety and reliability factor used in AGMA based design procedure. The method is illustrated with design of an automotive gear with a desired reliability, cost, and robustness. In this paper, we focus on the method rather than the results.

Keywords: Robust Design, Uncertainty, Design Factors, Reliability.

1 DESIGN OF GEARS

1.1 Traditional Gear Design Procedure & Design Factors

The traditional gear design is driven by the guidelines from AGMA [1], ISO, DIN, etc. These design procedures are similar with minor variations and contain a number of empirical design factors obtained from experiments and experience. These factors account for various uncertainties related to operating conditions and load, material and manufacturing processes, design parameters, precision and quality requirements, etc. A list of parameters used for steel spur gear design is given in Table 1 along with their dependence on various uncertainties related to geometrical design, material, manufacturing processing and operating conditions.

Factors of safety in bending and contact and the reliability factor are three key factors with significant influence on the design outcome and depend on the load conditions and material properties. These factors, in essence, account for uncertainty in load and material properties. AGMA in its standards states the following: “as design practices become more comprehensive, some influence factors have been removed from the unknown area of the “safety factor” and are introduced as predictable portions of the design method. The reliability factor, $K_R$, is an example” [1]. Hewitt [2] discusses selection of gear materials and estimation of reliability. He considers a statistical distribution of factor of safety having some part of the range less than unity and states that though purists may baulk at the concept of a safety factor less than unity, the procedure will be of engineering utility.

With the above in mind, the focus of the present work is on systematically accounting for variability in load and material properties to eliminate the dependence on these three factors in the AGMA design procedure, and yet lead to a robust design.
Table 1: List of Design Factors and their Dependence

<table>
<thead>
<tr>
<th>Factor</th>
<th>Material</th>
<th>Manuf.</th>
<th>Oper.</th>
<th>Const</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress factor for bending fatigue</td>
<td>✔</td>
<td>✔</td>
<td></td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>Stress factor for contact fatigue</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for temperature effects</td>
<td></td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for reliability</td>
<td>✔</td>
<td>✔</td>
<td></td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>Factor for overload</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for dynamic load</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for size effects</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for load distribution</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for hardness ratio</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for geometry</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for surface condition for pitting</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor for rim effects</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Elasticity constant</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor of safety in bending</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Factor of safety in contact</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>

1.2 Gear Design for Reliability and Robustness
Reliability based design of gears has received some attention in the literature. Stoker and colleagues [3] have studied the impact of errors of gear geometry on AGMA equations (analytical) and compared them with FEM results. Houser [4] studied the effect of accuracy requirements on the calculation of stresses using statistical design of experiments and Monte Carlo simulation techniques to quantify the effects of different manufacturing and assembly errors on root and contact stresses. Reliability is one of the empirical design factors used, which considers the statistical nature of failure of materials.

In recent years, considerable attention has been given to reliability-based design of gears. Zhang et al. [5] used techniques from perturbation method and reliability-based design theory and employed them for practical and effective method for the design of gear pairs. Aziz and Chassapis [6] explored application of the Stress-Strength Interference (SSI) theory within the context of a “Design for Reliability” for detailed gear design through evaluation the tooth-root strength with FEM-based verification.

Sensitivity analysis for reliability is discussed in References [7-12]. Yang and co-workers [7] studied reliability sensitivity based on reliability design theory, the Edgeworth series method, and the sensitivity analysis method. They discussed the reliability sensitivity of the cylindrical gear pairs with non-Gaussian random parameters and presented a numerical method for reliability sensitivity based design. Sun and colleagues [8] used limit state theory and studied the sensitivity of gear reliability with the change of various random factors. In this method a procedure is developed to find sensible random factors that have larger influence on gear reliability for the convenience of strict control of gear design, manufacturing, use and future maintenance in order to meet the requirement for gear reliability. The work of Zhang and colleagues [9] proposed a method to calculate the reliability and the reliability sensitivity of gear pairs. Zhang and Cui [10] worked on reliability design, a Kriging approximation model and an optimization method to build an optimization model, which can be used with the objective of minimizing the volume and cost of the gears while meeting the design criteria including reliability.

Robust design of gears has received less attention. MacAldener [11] has proposed methods for robust design of gears addressing slender teeth gears. Kulkarni, and colleagues [12] have used the compromise Decision Support Problem construct for robust design of gears and have explored the design and material space for gear design.

2 ROBUST DESIGN UNDER UNCERTAINTY
Papers by Bryne [13] and Taguchi and colleagues [14] represent first efforts creating robust designs. They introduce a method to minimize the effects of uncontrollable parameters during design. Ross, and colleagues [15] used the Taguchi loss function to make a design more tolerable to the model variations. Other researchers [16-19] use optimization to minimize the variation of input parameters to obtain designs with lower sensitivity of performance to design parameters. They propose a robust design optimization with Taguchi loss function as an objective function subject to the model constraints. Implementing this, the constant and variable sensitivity from controllable and uncontrollable parameters are respectively minimized using nonlinear programming. Padulo [20] discusses two approaches for robust optimization in which design parameters are stochastic.

Robust design involves achieving the system performance while minimizing the sensitivity of performance objectives with respect to the design variables. Achieving robustness in the presence of uncertainty may lead to designs that differ substantially from those anchored in traditional optimization that inherently does not consider uncertainty. The objective is to achieve ‘satisficing’ solutions that provide good performance despite the presence of uncertainty, as opposed to solutions that are optimum in a narrow range of conditions but perform poorly when the conditions change slightly.

Mistree and co-workers have proposed the compromise Decision Support Problem (cDSP) construct for robust design with multiple goals [21-23]. The cDSP is a hybrid formulation based on mathematical programming and goal programming concepts. It enables the construction of different practical scenarios in a multi-objective formulation. By giving appropriate weight to different goals, the compromises among
them can be explored. The dDSP minimizes the difference, \(d_i\) between the desired (the target \(G_i\)) and the achieved (\(A_i(x)\)), value of a goal. The difference between these values is the deviation value, which represents overachievement, \(d_i^+\), or underachievement \(d_i^-\) of each goal. The details of dDSP can be found in References [21 - 23].

The dDSP construct can be used to perform robust design by simultaneously maximizing the expected performance and minimizing the deviation from the expected performance. Within a dDSP, these two objectives are often treated as two separate goals that are traded against one another. Typically, robust design problems result in Pareto families of solutions for which it may be impossible to improve one objective without worsening another. In the following section, we discuss the dDSP based robust design formulation for gears.

3 ROBUST GEAR DESIGN WITH dDSP – FORMULATION

3.1 AGMA Based Design and Assumptions

During a preliminary analysis, it was observed that minimizing the contact factor of safety is more critical than the bending factor of safety, as it generally tends to be higher. Hence we concentrate in this paper in dealing with the factor of safety in the contact region. This is seen to be a general overriding factor and would also result in a higher factor of safety in bending. As in Reference [1], the induced stress in the contact is given by:

\[
\sigma_{\text{induced}} = Z_E \frac{W^t K_o K_f K_r K_H Z_R}{m N b Z_I} \]

(1)

Where \(Z_E\) is the elastic coefficient, \(W^t\) is the tangential load, \(K_o\), is the overload factor, \(K_f\) is the dynamic load factor, \(K_H\) is the load distribution factor, \(K_r\) is the size factor, \(m\) is the gear module, \(N\) is the number of teeth, \(b\) is the face width, \(Z_R\) is the surface condition factor for pitting resistance, and \(Z_I\) is the geometry factor. The methods for computing various factors can be found in References [1] and [24]. The tangential load \(W^t\)is given by:

\[
W^t = T/r_{\text{HPSTC}}
\]

(2)

where \(T\) is the applied torque and \(r_{\text{HPSTC}}\) is the radius of highest point of single tooth contact [24].

The design parameters are selected such that:

\[
\sigma_{\text{induced}} \leq \sigma_{\text{allowable}}
\]

(3)

where the allowable contact stress is given by:

\[
\sigma_{\text{allowable}} = \frac{0.6 S_U Z_N Z_W}{S_C Y_b K_R}
\]

(4)

where \(S_U\) is the material ultimate tensile strength, \(Z_N\) is a factor for stress cycle, \(Z_W\) is a factor for hardness ratio, \(Y_b\) is a factor for temperature effect, \(K_o\) is the factor for reliability and \(S_C\) is the factor for safety for contact analysis.

In the above expression, the reliability factor \(K_o\) “accounts for the effect of the normal statistical distribution of failures found in materials testing”. The factor \(S_C\) is “an additional safety factor should be considered to allow for safety and economic risk considerations along with other unquantifiable aspects of the specific design and application (variations in manufacturing, analysis etc.)” [1].

Our aim in this study is to eliminate the need for the factors \(K_o\) and \(S_C\).

3.2 Uncertainty in Load and Material Properties

AGMA provides an expression for \(K_o\) as a function of desired reliability and recommends methods for the selection of factor of safety. In this work we set \(K_o = 1\) and express factor of safety as below by equating the induced stress (Eq. (1)) with the allowable stress (Eq. (4)) and used this in constraints and goals. Through this, we have eliminated the use of two “design factors” from AGMA. By setting the factor of safety in bending to one, we have eliminated the third “design factor.”

\[
S_C = \frac{0.5 \times S_U \times Z_N \times Z_W}{Y_b \times Z_E} \sqrt{\frac{m \times N \times b \times Z_I}{W^t \times K_o \times K_f \times K_s \times K_H \times Z_R}}
\]

(5)

In order to account for uncertainty in material properties and load related economic risk, we assume that \(S_U\) and \(T\) have random Gaussian distribution with respective standard deviations \(\sigma_{S_U}\) and \(\sigma_T\). Accounting for uncertainty in material properties would eliminate the need for the reliability factor \(K_o\) discussed in section 3.1; and accounting for the uncertainty in the load would effectively help set the factors of safety in contact and bending to one, effectively eliminating the need for these two “design factors” that are to be either taken from AGMA guidelines or experience.

The standard deviation of \(S_C\) from Eq. (5) can be written as:

\[
\sigma_{S_C} = \sqrt{\left(\frac{\partial S_C}{\partial S_U}\right)^2 \times \left(\sigma_{S_U}\right)^2 + \left(\frac{\partial S_C}{\partial T}\right)^2 \times \left(\sigma_T\right)^2}
\]

(6)

This can be reduced to:

\[
\sigma_{S_C} = \frac{C}{\sqrt{T}} \times \sqrt{\left(\sigma_{S_U}\right)^2 + \frac{S_U^2 \times \sigma_T^2}{T^2}}
\]

(7)

where \(C\) can be derived from Eqs. (2), (5) and (6).

Under ideal conditions, when there is no uncertainty in load and material properties, a value of \(S_C\) of marginally more than one should suffice. A factor of safety less than unity would lead to failure.
With an assumption that the factor of safety $S_c$ also has a Gaussian distribution, one can define reliability \[ 2 \] in terms of design capability index $K$ \[ 23 \] as:

$$K = \frac{S_{c\text{mean}}}{\sigma_{S_c}}$$ \hspace{1cm} (8)

$K$ is related to the reliability as given in Table 2.

### Table 2: Reliability as a function of design capability index

<table>
<thead>
<tr>
<th>$K$</th>
<th>Reliability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.65</td>
<td>95.00</td>
</tr>
<tr>
<td>2.00</td>
<td>97.73</td>
</tr>
<tr>
<td>2.50</td>
<td>99.38</td>
</tr>
<tr>
<td>3.00</td>
<td>99.87</td>
</tr>
<tr>
<td>3.50</td>
<td>99.97</td>
</tr>
<tr>
<td>3.72</td>
<td>99.99</td>
</tr>
</tbody>
</table>

### 3.3 The cDSP Formulation

In the current work, we are interested in the design of a pinion for first gear reduction for a compact sized automobile. Following the example in Reference 21, the cDSP for robust design of gear is formulated as given below.

**Given**

It is required to design a pinion of a commercial spur gear system of AGMA precision no. 8 for torque of 113 Nm @ 4500 rpm with speed reduction of 3.5 (G=3.5) having a target for reliability of 99.99% but not less than 95%. The gear teeth are having pressure angle = 20° and they are cut using rack cutter. The expected fatigue life is of $10^9$ cycles. The materials for gear are available in the range of tensile strength of 800 MPa to 1600 MPa. Standard deviations in torque $T$ and ultimate tensile strength $S_{U\text{allowable}}$ are $30$ Nm and $50$ MPa respectively.

**Find**

Find the design variables that meet the design requirements. Here we consider the key design variables as:

- module ($m$) in mm,
- number of teeth ($N$),
- and face width ($b$) in mm. We are using these geometry variables along with
- material strength ($S_c$) in MPa

The description of module, number of teeth and face width can be found in Reference 24.

**Satisfy**

**Bounds on design variables:**

- B1: $4 \leq m \leq 8$ (mm)
- B2: $18 \leq N \leq 40$
- B3: $40 \leq b \leq 80$ (mm)
- B4: $800 \leq S_c \leq 1600$ (MPa)

**Design constraints:**

- C1: Minimum face-width*: $b \geq 3m$
- C2: Maximum face-width*: $b \leq 5m$
- C3: Maximum limit on center distance*: $d = m(1+G)N/2 \leq 300$ mm
- C4: Bending stress induced: $\sigma_{b\text{allowable}} - \sigma_{b\text{induced}} \geq 0$ ;
  Taken from reference 24 with factor of safety and reliability factor set to 1.
- C5: Contact stress induced: $\sigma_{c\text{allowable}} - \sigma_{c\text{induced}} \geq 0$ ;
  from equations (1) and (4)
- C6: Minimum contact ratio*: $R_x \geq 1.4$
- C7: Maximum contact ratio*: $R_x \leq 1.8$
- C8: Minimum reliability: $K \geq 1.65$ ; (corresponding to minimum permissible reliability of 95%; see Table 2).

**Goals:**

- G1: Maximize reliability $K$, given in Eq. (8), with a target value of 3.72
- G2: Minimize $\alpha_{S_c}$ of Eq. (7) (with a target value of 0.1 Maximize robustness)
- G3: Minimize cost $C$ with target cost is INR 57

Total cost consists of material cost and manufacturing cost. The expression for cost is given by Eq. (9), where the first term in the square brackets accounts for the manufacturing cost per unit weight, the second term accounts for the material cost per unit weight and $\rho$ is the density, taken as 7800kg/m$^3$ for this study.

$$COST = [540 + \left( \frac{S_c - S_{U\text{MIN}}}{S_{U\MAX} - S_{U\MIN}} \right)^{1.5}] \times \pi \rho b \left( \frac{mN}{2} \right)^2$$ \hspace{1cm} (9)

**Minimize**

The objective of the cDSP is to minimize the deviation function. The deviation function is constructed as shown below, where the system goals and constraints are normalized as per Reference 21.

$$Z = \{(d_i^+ + d_i^-), (d_i^+ + d_i^+), (d_i^+ + d_i^+)\}$$ \hspace{1cm} (10)

The deviation variables are selected such that $d_i^+ \cdot d_i^- = 0$ for $i = 1,...,3$ and $d_i^+, d_i^+ \geq 0$ for $i = 1,...,3$. As we are minimizing for multiple goals (maximization of
reliability \((K)\) is considered as minimization of \((1-K)\), the negative deviation is always 0 and Eq. (10) simplifies as

\[
Z = \{d_1^+, d_2^+, d_3^+\}.
\]

(11)

Archimedean approach [21] is used to construct the deviation function and it is solved using the software DSIDES\(^1\)

\[
Z(d^+, d^-) = \sum W_i d_i^+ \quad i = 1 \ldots 3
\]

\[
\sum W_i = 1 \quad \text{and} \quad W_i \geq 0 \quad \text{for all } i.
\]

(12)

(13)

4 RESULTS & DISCUSSION

The gear design problem described in the earlier section is formulated in the cDSP construct is solved under different scenarios of weights for goals and the outcome is discussed in this section. Further, a comparison with traditional gear design was made. To show the effectiveness of the method in dealing without the three design factors of AGMA

4.1 Design Scenarios

In order to explore the design space and evaluate the proposed method, various scenarios of target goals are studied by assigning different weights to the goals as shown schematically in Fig. 1. It can be seen that these goals are generally conflicting and compromise between these is to be explored for making design decisions. This study has primarily looked at seven scenarios as given Fig. 1. The corresponding weights are given in Table 3.

4.2 Discussion of Scenarios

Design parameters and goal achievement for different scenarios as discussed above are evaluated using the cDSP construct. Goal achievement for different scenarios is show in Fig. 2 below. The goal achievement values are normalized representing a value of 1 correspond to achieving lowest cost, highest reliability and highest robustness (low \(\sigma_g\)).

Scenarios S1, S4 and S5 have unique goals and the outcome represents the same. It can be seen here that for S4 and S5 cases, where robustness has to be maximized or cost has to be minimized without targets on reliability, the reliability has hit the lower bound. When reliability is the only target, the cost and robustness goal achievement has stayed at lower levels. This is in the expected lines.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Reliability</th>
<th>Robustness</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S2</td>
<td>0.5</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>S3</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
</tr>
<tr>
<td>S4</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>S5</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>S6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>S7</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
</tbody>
</table>

Table 3: Weights of goals for the scenarios considered

Scenario S4, S5 and S6 converge essentially to the same goal achievement. This is due to the solution reaching the lower limit of reliability (used as constraint) allowing for satisfaction of robustness and cost goals together.

Scenarios S2 and S3 represent compromise situations between reliability and robustness, and reliability and cost respectively. It can be seen that in both these cases, the reliability achieved is reasonably high, indicating its dominance. However, of these two, in scenario S2, the robustness and cost goals are better achieved as compared to S3. However this is at a cost of lower reliability. Scenario S6, where reliability is not a goal, the achievement has reached the same solution as S4 and S6 and this is due to the same reasons discussed earlier – the solution hitting the constraint on reliability. Scenario S7, where all three goals are of equal weight, has achieved target reliability and also performed well on other two counts. We can see that scenarios S2 and S7 lead to the best solutions as reliability for these two is close to the maximum value and simultaneously cost and variation in factor of safety are lower. Of these two, S2 represents a situation, which has minimum variation in factor of safety, comparatively higher reliability and lower cost. If higher reliability is desired, S7 can be the best scenario to go for.

The minimum value for \(\sigma_g\) achieved during these studies is 0.183 while the target is 0.10, indicating that this value was practically unachievable with a mean factor of safety greater than 1 (satisfying constrains C2 and C3) and simultaneously keeping the reliability greater than 95% (constraint C8). The lowest mean factor of safety observed is as low as 1.3. This is in contrast to the recommended factor of safety by AGMA of 2. Though it is not clear up to what extent AGMA considered uncertainty in parameters, it can now be easily quantified and

taken into account while designing. The current design can be seen to be robust as we try to minimize the standard deviation of the factor of safety.

4.3 Comparison with original AGMA based design
Scenario S2 and S7 are considered for validation with AGMA based designs accounting for the values for reliability factor and factor of safety as recommended. The reliability factor, as recommended by AGMA is given by:

\[
(15)
\]

The reliability factor was taken as per the value obtained in the earlier studies and the factor of safety as 2. A design following the AGMA prescribed methods was followed for the same problem and the design parameters were compared. The outcome in terms of design parameters was found to be reasonably close to the proposed method as seen from Tables 4 and 5. The cost of the component when AGMA procedure is used is found to be greater than the present method.

It has to be noted that the comparison of current method with AGMA would depend on the values of standard deviation for torque and ultimate tensile strength of the materials. Here, the authors have taken these from their past experience and they have no details of ranges considered by AGMA while deriving these factors. The good agreement with the outcome shows the possible the utility of the proposed method, while more studies with actual data have to be carried out.

Table 4: Comparison for S2 with AGMA based design

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>AGMA</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (mm)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b (mm)</td>
<td>67.3</td>
<td>62.9</td>
</tr>
<tr>
<td>N</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>UTS (MPa)</td>
<td>1550</td>
<td>1600</td>
</tr>
<tr>
<td>COST (INR)</td>
<td>192</td>
<td>178</td>
</tr>
</tbody>
</table>

Table 5: Comparison for S7 with AGMA based design

<table>
<thead>
<tr>
<th>Design Variable</th>
<th>AGMA</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>m (mm)</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>b (mm)</td>
<td>39.9</td>
<td>40</td>
</tr>
<tr>
<td>N</td>
<td>29</td>
<td>26</td>
</tr>
<tr>
<td>UTS (MPa)</td>
<td>1590</td>
<td>1550</td>
</tr>
<tr>
<td>COST (INR)</td>
<td>277</td>
<td>215</td>
</tr>
</tbody>
</table>

5 SUMMARY
In this paper a method is presented for the robust design of gears based on the compromise Decision Support Problem construct. Three important design factors (factors of safety in bending and contact and reliability factor) used in standard AGMA based design procedures are eliminated through formal introduction of uncertainty in the magnitude of load and material properties. The outcome of the solution is in line with that which is expected based on experience and are in for select cases are in close agreement with what can be obtained using AGMA. The solution thus obtained reinforces the possibility of systematically reducing the number of empirical design factors. As we get more and more information about material properties and manufacturing processes as well as a knowledge of loads, the proposed method can be used improve the design procedures with reduced number of factors from the AGMA based design without lengthy experimentation to modify the factors under new conditions. This would save a significant amount of time of experimentation for the industry and also help make continual changes to design when additional information on load variability and material variability become available.

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