POLICY DESIGN FOR SUSTAINABLE ENERGY SYSTEMS CONSIDERING MULTIPLE OBJECTIVES AND INCOMPLETE PREFERENCES

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ABSTRACT
The focus of this paper is on policy design problems related to large scale complex systems such as the decentralized energy infrastructure. In such systems, the policy affects the technical decisions made by stakeholders (e.g., energy producers), and the stakeholders are coordinated by market mechanisms. The decentralized decisions of the stakeholders affect the sustainability of the overall system. Hence, appropriate design of policies is an important aspect of achieving sustainability. The state-of-the-art computational approach to policy design problem is to model them as bilevel programs, specifically mathematical programs with equilibrium constraints. However, this approach is limited to single-objective policy design problems and is based on the assumption that the policy designer has complete information of the stakeholders’ preferences. In this paper, we take a step towards addressing these two limitations. We present a formulation based on the integration of multi-objective mathematical programs with equilibrium constraints with games with vector payoffs, and Nash equilibria of games with incomplete preferences. The formulation, along with a simple solution approach, is presented using an illustrative example from the design of feed-in-tariff (FIT) policy with two stakeholders. The contributions of this paper include a mathematical formulation of the FIT policy, the extension of computational policy design problems to multiple objectives, and the consideration of incomplete preferences of stakeholders.

Keywords: Sustainability, energy policy, feed-in-tariff policy, Game theory, Nash equilibrium, market models

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{tot,norm}$</td>
<td>Normalization factor for quantity</td>
</tr>
<tr>
<td>$\alpha, \beta$</td>
<td>Market constants</td>
</tr>
<tr>
<td>$NPV$</td>
<td>Net present value</td>
</tr>
<tr>
<td>$NPV_{max}$</td>
<td>Maximum net present value</td>
</tr>
<tr>
<td>$OP$</td>
<td>Operation and maintenance cost</td>
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<tr>
<td>$CI$</td>
<td>Capital investment</td>
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<tr>
<td>$CI_{max}$</td>
<td>Maximum capital investment</td>
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<tr>
<td>$PC$</td>
<td>Policy cost</td>
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<tr>
<td>$PC_{max}$</td>
<td>Maximum policy cost</td>
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<tr>
<td>$p_m$</td>
<td>Market price</td>
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<tr>
<td>$c$</td>
<td>Cost of electricity</td>
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<tr>
<td>$E_q$</td>
<td>Set of quantities satisfying the equilibrium constraints</td>
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<td>$d$</td>
<td>Market demand</td>
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<tr>
<td>$j$</td>
<td>Discount rate</td>
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<tr>
<td>$w_{11}$</td>
<td>Policy maker’s weight for quantity</td>
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<tr>
<td>$w_{12}$</td>
<td>Policy maker’s cost preference</td>
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<tr>
<td>$w_{21}$</td>
<td>Weight for stakeholder 1’s net present value</td>
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<tr>
<td>$w_{21}$</td>
<td>Weight for stakeholder 1’s capital investment</td>
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<tr>
<td>$w_{22}$</td>
<td>Weight for stakeholder 2’s net present value</td>
</tr>
<tr>
<td>$w_{22}$</td>
<td>Weight for stakeholder 2’s Capital Investment</td>
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</table>

1. INTRODUCTION – POLICY DESIGN FOR SUSTAINABLE ENERGY SYSTEMS
Traditionally, the emphasis in the engineering design research has been on problems where the design space is entirely under the control of designers. However, there is an increasing importance of large-scale complex systems whose designs are not directly controlled by designers, but emerge out of the independent decisions of self-interested stakeholders coordinated through market-based mechanisms. Consider the example of a smart electric grid, a large-scale complex system consisting of a wide range of decision makers including consumers, utilities, micro-grid operators, and the other participants of the distribution infrastructure. The distribution infrastructure includes distribution lines and cables, transformers, control devices, distributed generation devices, transformers, control devices, distributed generation devices,
In this paper, we present a formulation and a simple solution approach for addressing these two limitations of approaches for policy design. The formulation is based on an extension of the well known mathematical programs with equilibrium constraints (MPECs), games with vector payoffs, and Nash equilibria of games with incomplete preferences. We present the approach using an illustrative example from the design of feed-in-tariff (FIT) policy. The example problem is limited to two stakeholders to retain the ability to plot the decisions and rational reaction sets on 2D plots. The key contributions of this paper include extension of computational policy design problems to multiple objectives, the consideration of incomplete preferences of stakeholders, a mathematical formulation of the FIT policy.

The paper is organized as follows. In the following section, a detailed review of the literature is presented. An overview of the FIT policy is presented in Section 3. The mathematical tools used in the proposed approach, including MPEC and games with vector payoffs, are presented in Section 4. Mathematical formulation of the multi-objective FIT policy with incomplete preferences is presented in Section 5. Results of the illustrative example are presented in Section 6. Discussion of limitations and future research opportunities are presented in Section 7.

2. REVIEW OF RELEVANT LITERATURE

2.1. Modeling stakeholder decisions using non-cooperative games

The natural framework for analyzing systems that involve multiple independent decision-makers is non-cooperative game theory [1]. Non-cooperative games have been used in engineering design, primarily as a way to represent decentralized design scenarios [2, 3] where designers are modeled as decision-makers. The designers’ decisions are in equilibrium if none of the designers can unilaterally improve their payoff by changing their own decisions. This equilibrium is referred to as the Nash equilibrium. Current research on non-cooperative game theory within engineering design is focused on determining the Nash equilibria and their stability properties.

One of the widely adopted approaches for finding Nash equilibria is based on formulating the problem as a complementarity problem [4, 5] and using the first order necessary conditions of optimality of the individual stakeholders’ decisions. The complementarity problem is a special case of a variational inequality problem [5-7]. The complementarity models for Nash equilibria have been used in a number of applications related to modeling of markets [8]. For example, Hobbs [9] presents a model of bilateral markets with imperfect competition between electricity producers using linear complementarity models. Gabriel and co-authors [10-13] model a natural gas equilibrium model with different types of market participants including producers, storage operators, pipeline operators, marketers, and consumers.

2.2. Policy-design as a bilevel problem

While modeling of equilibria between stakeholders in a market is an important problem, the goal from a policy-design
standpoint is to design the Nash equilibria by influencing stakeholders’ decisions. This problem of designing equilibria can be viewed as a higher-level problem with design variables (e.g., incentives or penalties) that can be used to modify the Nash equilibria of the lower-level equilibrium problem. These design problems represent a special class of bilevel programs [14]. Within mathematical programming literature, such bilevel problems are called mathematical programs with equilibrium constraints (MPECs) [15]. Ye [16, 17] presents necessary and sufficient conditions for optimality for bilevel programs and MPECs.

MPECs are challenging because the optimality conditions in the lower level problems lead to combinatorial issues, and the potential lack of convexity and/or closedness of the feasible region [15]. A number of specialized algorithms have been developed to address these challenges of MPECs [15, 18-20]. Examples of the algorithms include piecewise sequential quadratic programming, penalty interior-point algorithm, implicit function based approaches, and smooth non-linear programs [18]. Some of these algorithms are implemented in commercial platforms for optimization such as GAMS and Matlab [21]. Applications of MPEC include electricity markets [22, 23], highway tax policy design [24], and critical infrastructure planning [25].

2.3. Gap in the literature

The existing work on designing policies using bilevel programming techniques has two main limitations. The first limitation is that the problems are modeled as single-objective problems. Recently, there have been some efforts within the mathematical programming area on extending the MPEC formulation to multi-objective optimization problems with equilibrium constraints (MOPEC) [26, 27]. The current work in that direction is focused on deriving the necessary conditions for optimality [28, 29]. However, such formulations have not yet been utilized for policy design problems. In this paper, we discuss the application of MOPEC to problems involving policy design for sustainability.

The second limitation of the existing literature is that it is assumed that the higher level policy designer has complete knowledge of the preferences of the lower-level decision makers. The common assumption is that the stakeholders are profit-maximizing firms and their only objective is to maximize their profits. However, the stakeholders may have multiple objectives. Consider an example of a policy decision at the federal level, which affects the decisions made by local (or state) policy makers. In this case, the federal policy design is the upper-level problem and the local policy design is the lower level problem in MPEC, whose goal is not simply profit maximization. Even the profit maximizing firms have objectives that cannot be directly quantified in terms of profit. Examples of such objectives include service quality, brand recognition (through reduced green-house gas emissions), and community service. Even in cases where the multi-objective nature is acknowledged, it is implicitly assumed that the policy decision-maker knows the stakeholders’ tradeoffs in advance, allowing the lower-level decisions to be modeled as single-objective optimization problems. Hence, it is assumed that the stakeholders’ objectives can be combined into a single objective function (such as a utility function). This single objective function satisfies the completeness axiom of vonNeumann and Morgenstern’s utility theory [30] and can be used to compare all alternatives. However, this assumption can be invalid in three scenarios which are particularly relevant in real policy design problems [31, 32]. First, the policy decision maker may not have complete information about the preferences of the individual decision makers. Second, the lower-level decision makers may represent groups of individuals (e.g., committees), leading to incomplete social preferences. Third, the decision makers may be indecisive, and hence, unable to rank all combinations of alternatives in a multi-objective scenario [33].

To address the incomplete nature of preferences, vonNeumann and Morgenstern’s utility theory has been extended to utility theory with incomplete preferences [34-37]. While existing work on utility theory with incomplete preferences is focused on modeling the decisions, there is limited understanding of strategic interactions between players with incomplete preferences. Bade [38] shows that the Nash equilibria for any game with incomplete preferences can be characterized in terms of certain derived games with complete preferences. Additionally, if the players’ preferences are concave, the Nash equilibria can be determined from derived complete games by a simple linear procedure. The author [38] discusses the Nash equilibrium of a game where a) each decision maker has multiple objectives, b) decision makers are able to rank alternatives based on each objective individually, and c) the decision makers are unable to make tradeoffs among different objectives. Such games are also referred to as games with vector payoffs [39] or multi-objective games [40, 41]. The equilibria of games with vector payoff are referred to as Pareto equilibria [42, 43].

In this paper, we consider the multi-objective nature of the policy design problem and the incomplete preferences of stakeholders. We illustrate a framework based on MOPEC and games with vector payoffs using Feed-In-Tariff policy design problem.

3. SUSTAINABLE ENERGY INFRASTRUCTURE AND THE FEED-IN-TARIFF POLICY

3.1. Energy policy and the interplay between policy design and engineering design

With the increasing use of small-scale energy generation from renewable sources and increasing deregulation of the energy sector, an alternative paradigm of energy generation and distribution is emerging and leading towards “smart grid architecture”. In a decentralized infrastructure, different stakeholders act as decision makers, and the overall system-level performance is dependent on the individual decisions. For example, the consumers can play an active role as energy producers for actively managing their demand. They make decisions on a) which technologies to invest in, b) how much energy to generate, c) how much energy to buy and from whom, d) how much energy to sell and e) how much to participate in actively managing their load demand [44]. Other stakeholders include power producers (e.g., utility companies), grid operators, transmission companies (TRANSCO), distribution companies (DISTCO) and regulators (e.g.,
The policy design problem is driven by a number of social, environmental, technical, and economic objectives [45]. The technical objectives include replacing fossil fuel generating plants with renewables, minimization of system losses, maintaining required stability/security/reliability, avoiding unbalance conditions, meeting power quality requirements, peak shaving, targeting high efficiency systems, innovation and early adoption of technologies and meeting the energy needs. The environmental objectives are minimization of green house gas emissions and hazardous materials. Economic objectives include minimization of policy costs and ratepayer impact. Social objectives include job creation, economic development, meeting long term energy requirements, policy transparency, fairness and quality of life.

To achieve these objectives, different policies can be adopted at the federal, state, local, and utility levels. The policy options include incentives to investment, guidelines for energy conservation, taxation and other public policy techniques [46, 47]. Specific examples include emission taxes, incentives to non-polluters and renewable energy, incentive for demand response, emission cap-and-trade systems, emission intensity standards and regulations, and alternative allocations of emission rights to regions and sectors. In several counties, including Germany and Spain, one of the mechanisms which has been particularly successful in addressing environmental, reliability, and security issues associated with decentralized energy has been feed-in-tariff (FIT) policies [45]. FIT policies are discussed in the following section.

3.2. Overview of feed-in-tariff (FIT) policies

A feed-in-tariff is an energy supply policy that offers a guarantee of payments to renewable energy (RE) developers for the electricity they produce [48]. The objectives of these policies are to motivate the deployment of RE technologies and to increase renewable generation while reducing dependencies on fossil-fuels. FIT programs support decentralized infrastructure and motivate individuals along with companies to invest in renewable energy technologies. FIT can be designed by the utilities or the state government. Moreover, FIT can be designed to work in conjunction with other US state policies such as renewable portfolio standards (RPSs) and net-metering, and federal policies such as the Production Tax Credit (PTC) and the Investment Tax Credit (ITC).
4. MATHEMATICAL MODELS AND TOOLS USED

The decentralized decisions of the stakeholders can be modeled as a non-cooperative game between players (i.e., stakeholders) trying to achieve their own objectives. The outcome of the non-cooperative game is defined in terms of the Nash equilibrium. In the case of FIT policy, the equilibrium point is determined by the payment to the investors. Hence, the goal of the policy designer is to choose the payment (i.e., the parameters of the policy parameters) such that the equilibrium point represents the best system performance from a global standpoint.

Within game theory, such interactions between decision makers are referred to as the Stackelberg game where one of the players (in this case the policy maker) moves first and the rest of the players (the stakeholders) move based on the decision made by the player moving first. The Stackelberg game can be mathematically modeled as a bi-level optimization problem with policy design as the upper optimization problem and the stakeholders’ decisions as the lower-level Nash equilibrium problem. Mathematical Programs with Equilibrium Constraints (MPEC) is a mathematical framework for such bi-level problems with higher level optimization problem and lower level equilibrium problem. An overview of the MPEC framework is provided in Section 4.1. In Section 4.2, we discuss games with vector payoffs that are used for modeling the incomplete preferences of stakeholders.

4.1. Mathematical programming with equilibrium constraints (MPEC)

MPEC is a type of constrained nonlinear programming problem where some of the constraints are defined as parametric variational inequality or complementarity system. These constraints arise from some equilibrium condition within the system, and hence, are called equilibrium constraints. MPEC is applicable to a variety of problems in engineering such as optimal design of mechanical structures, network design, motion-planning of robots, facility location, and equilibrium problems in economics. Examples of problems of economic equilibrium where MPEC has been used include maximizing revenue from tolls on a traffic system, optimal taxation, and demand adjustment problems. Mathematically, a MPEC problem can be represented using two sets of variables, $x$ and $y$. Here, $x$ belongs to the upper-level problem and $y$ solves the lower-level equilibrium problem. The solution of $y$ depends on the value of $x$ chosen for the upper-level problem. The overall objective function $f(x, y)$ is minimized.

$$\min_{(x,y)} f(x, y)$$

subject to:

$$\begin{align*}
(x, y) &\in \Omega, \\
y &\in S(x)
\end{align*}$$

where $\Omega$ is the joint feasible region of $x$ and $y$; and $S(x)$ is a set of variational inequalities that represent the equilibrium problem. The function $f(x, y)$ represents a system-level function that quantifies the goodness of the solution. The set $S(x)$ corresponds to the feasible Nash space. The Nash equilibrium point can be formulated as a variational inequality using the first order necessary conditions for optimality such as Karush–Kuhn–Tucker (KKT) conditions. Solving the MPEC problems is challenging because of the non-linearities in the problem, non-convex feasible space, combinatorial nature of constraints, disjointed feasible space, and multi-valued nature of the lower equilibrium problem. Significant research efforts have been devoted to developing efficient algorithms for solving MPEC problems. These include piecewise sequential quadratic programming (PSQP), penalty interior-point algorithm (PIPA), implicit function-based approaches, and smooth sequential quadratic programming. Recently, few efforts has been carried out by Ye, Mordukhovich, and Bao et al. on deriving the necessary conditions for optimality of multi-objective problems with equilibrium constraints (MOPEC).

4.2. Games with vector payoffs

Games within which players can have several possibly conflicting objectives are called “games with vector payoffs” or “multi-objective games”. Due to the multi-objective nature of the decisions, the concepts of rational reactions and Nash equilibria need to be generalized. In the games with single objectives, a rational reaction (best reply) of a player to other players’ decisions is a point that maximizes his/her payoff. In the case of games with vector payoffs, a player’s rational reaction to the decisions of other players is a point of Pareto optimal solutions, also referred to as Pareto Best Replies. In single-objective games, the points of intersection of the rational reaction sets are the Nash equilibria. In games with vector payoff, the concept of Nash equilibrium is replaced by the concept of Pareto equilibria defined as the pairs of strategies which are Pareto best replies to one another.

In this paper, we use games with vector payoffs to represent situations where the policy designer knows the multiple objective functions of the players but does not have complete information about the players’ preferences for tradeoffs. In such situations, the policy designer cannot assign a single utility function to each player. Under certain conditions, games with vector payoffs can be reduced to games with single objectives by combining each player’s objectives using Archimedean weighting scheme. The reduced single objective game is called a tradeoff game or a derived game with complete information. In this paper, we convert the games with vector payoffs into multiple tradeoff games and solve the corresponding MOPECs to generate uncertainty bounds. The approach is discussed in detail in Section 5.

5. FORMULATION OF THE FIT POLICY DESIGN PROBLEM FOR SUSTAINABLE ENERGY SYSTEMS

The formulation of upper-level policy decision involves the identification of policy objectives, design variables, and constraints. There are a number of objectives that can be considered while designing FIT policy (see Table 1). As mentioned in Section 3.2, there are two classes of FIT policy – market independent and market dependent. In this paper we use the market-dependent design, specifically the premium price model to illustrate the approach. The lower-level stakeholders are modeled as profit-maximizing entities who invest in different technologies based on the incentives determined by the policymakers. Examples of design variables include the types and sizes of different generation technologies to invest in.
The stakeholder objectives are calculated in terms of the net-present value (NPV) of the all the expenses (capital investments and maintenance costs) and the payments received for the energy generated. One of the objectives of the stakeholders is to minimize NPV.

Table 1 - The objectives and design variables of the policy design problem

<table>
<thead>
<tr>
<th>FIT Policy Design Objectives [48]</th>
<th>Economic: job creation, economic development, economic transformation, stabilization of electricity prices, lower electricity prices, grow economy, revitalize rural areas, attract new investment, develop community ownership, develop future export opportunities</th>
<th>Environmental: clean air benefits, greenhouse gas emission reduction, preserve environmentally sensitive areas, manage waste streams, reduce exposure to carbon legislation</th>
<th>Energy Security: secure abundant energy supply, reduce long-term price volatility, reduce dependence on natural gas, promote a resilient system</th>
<th>Renewable Energy (RE) Objectives: rapid RE deployment, technological innovation, drive cost reductions, meet renewable portfolio standards, reduce fossil fuel consumption, stimulate green energy economy, barriers to renewable development</th>
</tr>
</thead>
</table>

| Design Variables                                      | Alternatives: 1) Premium price model: price per kWh; 2) Variable premium price model: premium price in access of the market price; 3) Percentage of retail value: % in access of the market price. | Payment differentiation based on technology and fuel type, project size, resource quality, resource quantity, and location. Other design variables include tariff depression, and inflation adjustment. |

5.1. Formulation of an illustrative FIT problem

To illustrate the approach for accounting for multiple policy design objectives and incomplete knowledge of stakeholders’ preferences, we present a simple example with two stakeholders. An overview of the decisions of the policy designer and each stakeholder is provided in Table 2. The policy designer has two objectives: a) to maximize the total quantity of energy generated by all the stakeholders (Q), and b) to minimize the cost of implementing the policy (C). The first objective is related to the policy goal of renewable energy penetration through the design of incentives and the second objective is an economic goal associated with most policies. The stakeholders’ decisions are driven by two objectives: maximization of the net-present value of their investment and the minimization of capital investment. The assumptions made in this model are listed in Section 5.1.1.

5.1.1. Assumptions

The problem formulation presented in this section is based on the following assumptions:
1) There is only one policy maker – In a real policy design problem, there are a number of entities involved in the policy making process. Different entities may have different objectives. An entity (such as environmental protection agencies) may have an objective to reduce CO₂ emissions, while another may be more interested in the economic impacts of a policy. Here, we consider that one policy maker is responsible for satisfying all the objectives and has complete control of the design variables.

Table 2 – Overview of the decisions made by the policy designer and the stakeholders

<table>
<thead>
<tr>
<th>Policy Designer</th>
<th>Objectives: Maximize total quantity generated by all stakeholders, Q Minimize the policy cost, C</th>
<th>Decision Variables: Premium price per unit energy generated, Δ Time period for which policy is implemented, T</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each stakeholder’s decision</td>
<td>Objectives: Maximize the net present value, NPV₁ Minimize the capital investment, CI₁</td>
<td>Decision Variables: Quantity of energy to be generated, q₁</td>
</tr>
</tbody>
</table>

2) There are two stakeholders – We consider only two stakeholders because it helps us in visualizing the rational reactions and the design space on 2D plots. An actual policy design problem generally consists of a large number of stakeholders. The model will be extended to n-stakeholders in the future.

3) Fixed premium-price FIT model payment option – As discussed in Section 3.2 there are many different models of FIT policies. In our model we assume a FIT policy where the payment is a fixed premium price (Δ). In other words, the investors are paid some set amount, Δ, above the market price of electricity.

4) The policy-maker can only control two decision variables – We set the policy designer’s decision variables as the premium-price of the policy, Δ, and the duration of the policy, T. These two variables are the primary design variables in a premium price model. In a real FIT design scenario, the premium price may be different for different types of technologies, and different size of generation facilities installed by the stakeholders.

5) Stakeholders can only control one decision variable – Stakeholders can only control the quantity of electricity being generated (q₁). Based on the duration of the policy, the premium-price and the market demand, each stakeholder decides upon the quantity to generate.

6) The policy options are not dynamic – In many FIT policies the policy options (decision variables) can vary with time. For example, the premium price may be high during the initial period to encourage investment but may be reduced over time. Alternatively, the premium price may increase with time to account for inflation. In this paper, we assume that Δ is fixed for the duration of the policy.

7) Incomplete information about preferences – It is assumed that the policy maker has complete information of the stakeholder’s individual objectives (maximization of net-present value and minimization of capital investment). However, the policy maker does not have complete information about how each stakeholder makes tradeoffs among the different objectives.
8) **Electricity market is modeled using Cournot Nash equilibrium** – In order to simplify how stakeholders respond to each other’s investments, we assume that the electricity market is modeled as Nash equilibrium of a Cournot competition game.

### 5.1.2. Simple model of the electricity market

The electricity market modeling literature [60] consists of two types of models for market equilibrium arising from profit maximizing participants: the Cournot equilibrium [61] and supply function equilibrium (SFE) [22]. Both concepts are based on the Nash equilibrium, but differ in the decision makers’ variables. In Cournot equilibrium model, the participants compete in quantity of energy produced, whereas in the supply function equilibrium model, the participants compete both in quantity and price.

Consider a simple example of two producers deciding on their production quantities \( q_1 \) and \( q_2 \) [62]. Assume that the market price is determined by the overall quantity produced \( q \) through a linear function: \( p(q) = (A - q) \) where \( A \) is a constant. The profit of each firm is given by: \( \pi_i(q_i, q_2) = (A - q_1 - q_2)q_i - c_i q_i \) where \( c_i \) is the cost of production for player \( i \). The resulting Nash equilibrium is given by:

\[
q_i^* = \frac{1}{3} \left( A - c_2 + 2c_1 \right) \quad \text{and} \quad q_2^* = \frac{1}{3} \left( A - c_1 + 2c_2 \right)
\]

Cournot equilibrium is more flexible and tractable because it results in a set of algebraic equations for the Nash equilibrium whereas SFE results in a set of differential equations [60]. Hence, the Cournot equilibrium model has attracted significant attention from the electricity market modeling community [9]. For a given price and duration of a policy, the stakeholders decide the quantities \( q_i \), which correspond to the Cournot equilibrium value based on the market and stakeholder preferences. The policy makers can achieve their objectives by designing the price and duration.

### 5.1.3. Details of the decisions made by stakeholders and the policy maker

The objectives, design variables, and constraints for the policy maker and the stakeholders are shown in Table 3. First, we consider the objectives of the stakeholders. The first objective is the net present value (NPV), which is the time series of cash inflow and outflow. The second objective is minimization of capital investment (CI). As previously stated, the policy maker has control over two variables, \( \Delta \), the premium-price of the policy (which is constant during the duration of policy implementation), and \( T \), the duration of the policy. We assume that the market price takes the form, \( p_m = \alpha - \beta(Q_{tot}) \), where \( Q_{tot} = \sum_{i=1}^{n} q_i \). Here, \( \alpha \) and \( \beta \) are two constants based on the energy market. For the simplified case of two stakeholders, \( Q_{tot} = (q_1 + q_2) \). We assume the cash inflow for stakeholder \( i \) is \( (p_m + \Delta)q_i \) and the outflow is the operation and maintenance cost (OP) along with capital investment (CI), see eqn. (1). Since capital investment is only made at the start of the investment this payment is not reoccurring and is not affected for varying time. Assuming that \( j \) is the discount rate, the net present value for stakeholder \( i \) is:

\[
NPV_i = \sum_{T}^{N=1} \frac{q_i(\alpha - \beta(q_1 + q_2) + \Delta)}{(1 + j)^N} \\
- \left( \sum_{N=1}^{T} OP \cdot q_i + CI \cdot q_i \right)
\]

The design variable for each stakeholder is the quantity, \( q_i \), which is constrained to be positive. A negative value of \( q_i \) would indicate that instead of generating energy, the stakeholder purchases it from the other stakeholder. However, we do not consider that scenario in this paper.

**Table 3 – Multi-objective policy design problem with multi-objective decisions of stakeholders. This is the most general problem**

<table>
<thead>
<tr>
<th>Upper-level policy design problem</th>
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<tbody>
<tr>
<td><strong>Objectives:</strong></td>
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<td></td>
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<tr>
<td><strong>Design variables:</strong></td>
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<td><strong>Constraints:</strong></td>
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<th>Lower-level stakeholders’ problems</th>
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<tr>
<td><strong>Stakeholder 1</strong></td>
</tr>
<tr>
<td><strong>Objectives:</strong></td>
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<td></td>
</tr>
<tr>
<td><strong>Design variable:</strong></td>
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<td></td>
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<td><strong>Constraints:</strong></td>
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</table>

The policy maker can control \( \Delta \), the non-variable premium-price of the policy, and \( T \), the duration of the policy. Based on these values, and the preferences from the stakeholders, the stakeholders reach equilibrium of the quantity being generated. If these preferences are known, the policy maker can maximize his/her own payoff based on these decisions. We assume that the policy maker has two objectives. The first objective is to minimize the policy cost (PC). The second objective is to maximize the total quantity of generation \( Q_{tot} \). We constrain the design variables where the premium-price of the policy is less than 0.2 S/KWh and the duration of the policy is less than or equal to twenty one years.

### 5.2. Approach for policy decision making under incomplete preferences of stakeholders

The approach adopted in this paper for solving the policy design problem listed in Table 3 is shown in Figure 3. There is a lack of algorithms for solving the general problem presented in the previous section. Hence, we follow a simple six step process. Step 1 is the formulation of the decisions of the policy decision maker and the stakeholders, as discussed in Section 5.1.3. The characteristics of this general problem are that the policy designer has multiple objectives and has incomplete information about the stakeholders’ preferences. We assume that the policy designer knows the different objectives of the stakeholders but has incomplete information about how they
Since the incomplete knowledge is assumed to be about tradeoffs between objectives, we use an Archimedean combination of the different objectives to define the payoffs, $\pi_i$, of the stakeholders in the games with complete information. The payoff functions are listed in equations (2) and (3) below. $NPV_{n_{\text{norm}}}$ and $CI_{n_{\text{norm}}}$ are used to normalize the net present value and capital investment. A set of games with complete information are obtained by choosing different values of the weights ($w_{211}$ and $w_{212}$).

$$\pi_1 = w_{211} \frac{NPV_1}{NPV_{n_{\text{norm}}}} + w_{221} \left[ 1 - \frac{CI_1}{CI_{n_{\text{norm}}}} \right]$$  \hspace{1cm} (2)

$$\pi_2 = w_{212} \frac{NPV_2}{NPV_{n_{\text{norm}}}} + w_{222} \left[ 1 - \frac{CI_2}{CI_{n_{\text{norm}}}} \right]$$  \hspace{1cm} (3)

where, $(w_{211} + w_{221}) = 1$ and $(w_{212} + w_{222}) = 1$.

Step 3 of the approach is to derive a set of MPECs from the MOPEC. This is achieved by taking weighted combinations of the policy designer’s objectives. The derived problem is shown in Table 5. The payoff of the policy designer is calculated as shown in Eq. (4). $Q_{\text{tot,norm}}$ and $PC_{\text{norm}}$ are used to normalize the values of total quantity and policy cost. Different values of weights ($w_{11}$ and $w_{12}$, with $w_{11} + w_{12} = 1$) result in different MPEC problems in Step 3.

$$\pi = w_{11} \frac{Q_{\text{tot}}}{Q_{\text{tot,norm}}} + w_{12} \left[ 1 - \frac{PC}{PC_{\text{norm}}} \right]$$  \hspace{1cm} (4)

Step 4 involves solving the derived MPEC problems using existing algorithms such as variations of NLP algorithms, and interior-point algorithms. The solution of the MPEC represents a single condition with given tradeoff between the policy designer’s objectives and complete information about stakeholders’ preferences. Some of these solutions within the set of all MPECs derived from a MOPEC (Step 2) are Pareto dominant. These Pareto dominant solutions are identified in Step 5. The resulting sets of solutions represent solutions of a MOPEC. The sets of solutions to the MPECs derived from the original policy design problem correspond to the uncertainty in the solution of the original design problem due to the incomplete preferences.

**Table 4 – Derived Problem 1: MOPEC formulation of the policy design problem with weighted objectives of the stakeholders**

<table>
<thead>
<tr>
<th>Upper-level policy design problem</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives:</strong></td>
<td>Minimize $PC = \sum q_i \Delta$</td>
</tr>
<tr>
<td></td>
<td>Maximize $Q_{\text{tot}} = \sum q_i$</td>
</tr>
<tr>
<td><strong>Design variables:</strong></td>
<td>$\Delta, T$</td>
</tr>
<tr>
<td><strong>Constraints:</strong></td>
<td>$0 \leq \Delta \leq 0.20 \text{ S/KWh}$</td>
</tr>
<tr>
<td></td>
<td>$T \leq 21 \text{ years}$</td>
</tr>
<tr>
<td>$q_i \in E_q$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower-level stakeholders’ problems</th>
<th>Stakeholder 1</th>
<th>Stakeholder 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective:</strong></td>
<td>Maximize $\pi_1(NPV_1, CI_1)$</td>
<td>Maximize $\pi_2(NPV_2, CI_2)$</td>
</tr>
<tr>
<td><strong>Design variable:</strong></td>
<td>$q_1$</td>
<td>$q_2$</td>
</tr>
<tr>
<td><strong>Constraints:</strong></td>
<td>$q_1 \geq 0$</td>
<td>$q_2 \geq 0$</td>
</tr>
</tbody>
</table>

**Table 5 – Derived Problem 2: MPEC formulation of the policy design problem with weighted objectives of the stakeholders**

<table>
<thead>
<tr>
<th>Upper-level policy design problem</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective:</strong></td>
<td>Maximize $\pi(PC, Q_{\text{tot}})$</td>
</tr>
<tr>
<td><strong>Design variables:</strong></td>
<td>$\Delta, T$</td>
</tr>
<tr>
<td><strong>Constraints:</strong></td>
<td>$0 \leq \Delta \leq 0.20 \text{ S/KWh}$</td>
</tr>
<tr>
<td></td>
<td>$T \leq 21 \text{ years}$</td>
</tr>
<tr>
<td>$q_i \in E_q$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lower-level stakeholders’ problems</th>
<th>Stakeholder 1</th>
<th>Stakeholder 2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objective:</strong></td>
<td>Maximize $\pi_1(NPV_1, CI_1)$</td>
<td>Maximize $\pi_2(NPV_2, CI_2)$</td>
</tr>
<tr>
<td><strong>Design variable:</strong></td>
<td>$q_1$</td>
<td>$q_2$</td>
</tr>
<tr>
<td><strong>Constraints:</strong></td>
<td>$q_1 \geq 0$</td>
<td>$q_2 \geq 0$</td>
</tr>
</tbody>
</table>
6. RESULTS FOR THE ILLUSTRATIVE PROBLEM

In this section, we present the results for the illustrative FIT policy design problem. As discussed in Section 5.2, the design problem with incomplete preferences is converted into multiple MOPECs and each MOPEC is converted into multiple MPECs. In Section 6.1, we discuss the results of a single MOPEC where the upper level designer has multiple objectives and the lower level stakeholders are associated with a single objective function ($\pi_i$) which is a linear combination of the two objectives with pre-defined weights ($w_{21i}$ and $w_{212}$). Section 6.2 contains a discussion of the results from Step 5 in Figure 3. In Section 6.2, the results of the original design problem, based on Step 6 are presented. There, the outcomes and the associated uncertainty resulting from the incomplete preferences are presented. The values of the parameters used for the results discussed in this section are shown in Table 6.

Table 6 – Values of parameters used in this model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{\text{norm}}$</td>
<td>$1.03 \times 10^6 \text{KWh}$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$0.69 \text{ $/KWh}$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$5 \times 10^{-6} \text{ $/KWh}$</td>
</tr>
<tr>
<td>$NPV_{\text{norm}}$</td>
<td>$3.5 \times 10^5 \text{ $/KWh}$</td>
</tr>
<tr>
<td>$OP$</td>
<td>$0.1 \text{ $/KWh}$</td>
</tr>
<tr>
<td>$CI_{\text{norm}}$</td>
<td>$1.04 \times 10^5 \text{ $/KWh}$</td>
</tr>
<tr>
<td>$CI$</td>
<td>$0.13 \text{ $/KWh}$</td>
</tr>
<tr>
<td>$PC_{\text{norm}}$</td>
<td>$2.08 \times 10^5 \text{ $/KWh}$</td>
</tr>
<tr>
<td>$f$</td>
<td>$0.06$</td>
</tr>
</tbody>
</table>

For the MPEC shown in Table 5, the Nash equilibrium condition (i.e., the equilibrium constraint for the upper-level problem) can be derived using the first-order optimality conditions for the stakeholders’ decisions. It is assumed that the payoff functions for both stakeholders are equivalent. The resulting quantity of generation for designer $i$ is:

\[
q_i = \frac{w_{22i} \cdot CI \cdot NPV_{\text{norm}}}{CI_{\text{norm}}} + w_{22i} \cdot CI - 2 \beta \cdot w_{22i} \cdot \frac{\sum_{n=1}^{T} \frac{1}{(1 + \gamma)^n}}{OP - \Delta + \beta q_i' - \alpha} + 2 \beta
\]

where $q_i$ is $q_2$ for $q_i = q_1$, and $q_1$ for $q_i = q_2$.

By solving the equilibrium conditions for the quantities, the equilibrium constraint is derived as:

\[
q_i = \frac{\alpha + \Delta - OP}{3\beta} - 2 \left( \frac{w_{22i} \cdot CI \cdot NPV_{\text{norm}} + w_{21i} \cdot CI}{3\beta \cdot w_{21i} \cdot \frac{\sum_{n=1}^{T} \frac{1}{(1 + \gamma)^n}}{CI_{\text{norm}}}} \right)
\]

\[
+ \frac{w_{22i} \cdot CI \cdot NPV_{\text{norm}} + w_{21i} \cdot CI}{3\beta \cdot w_{21i} \cdot \frac{\sum_{n=1}^{T} \frac{1}{(1 + \gamma)^n}}{CI_{\text{norm}}}}
\]

(6)

To demonstrate the feasible ranges of equilibrium quantities ($q_i$), we show the region within which Nash equilibria can possibly lie for $0 \leq \Delta \leq 0.20$ $$/KWh$ and $0 \leq T \leq 21$ in Figure 4. The shaded regions represent the feasible region for the upper-level problem, as constrained by the equilibrium constraint. Four different levels of uncertainty of stakeholders’ preferences are shown. Figure 4(a) shows nearly complete uncertainty with $w_{211}$ and $w_{212}$ both within the range $[0.1, 1.0]$. Figure 4(b) represents a reduced uncertainty about $w_{211}$ and $w_{212}$ where both weights are within the range $[0.2, 0.4]$. Figure 4(c) shows another scenario of reduced uncertainty about $w_{211}$ and $w_{212}$. Here, the weights are in the range $[0.7, 0.8]$. Note that despite the reduction in the width of the interval representing uncertain payoffs, the size of the region within which the equilibrium can lie is larger than in Figure 4(b). The uncertainty in the preferences of the stakeholders may not be uniform across stakeholders, as shown in Figure 4(d), where $w_{211}$ is within $[0.7, 0.8]$ and $w_{212}$ is within $[0.6, 0.7]$. In this case, the space within which the equilibrium points lie is not symmetric.

6.1. Results for the policy design with precise knowledge of stakeholder preferences

Consider a derived MOPEC in Step 2 where the stakeholders are only interested in maximizing their net present value ($NPV_i$). This refers to the condition where $w_{211} = w_{212} = 1.0$ and $w_{221} = w_{222} = 0.0$. Since the derived MOPEC assumes complete knowledge of preferences of stakeholder, the policy maker can choose the decision variables which maximize the policy-level objectives. In this case, Eq. (5) reduces to Eq. (7), and Eq. (6) reduces to Eq. (8). The resulting quantity for stakeholder $i$ is:

\[
q_i = \frac{CI}{\sum_{n=1}^{T} \frac{1}{(1 + \gamma)^n}} + OP + \beta q_i' - (\alpha + \Delta)
\]

where,

\[
q_i' = \frac{q_2}{q_1}
\]

This equilibrium condition relates a stakeholder’s decision ($q_i$) to the other stakeholder’s decision ($q_i$), and is also referred to as the rational reaction set (RRS). This condition results in
two equations in \( q_1 \) and \( q_2 \). On solving the two equations, the quantity produced by designer \( i \) is determined to be:

\[
q_i = \frac{(\alpha + \Delta) - OP}{3\beta} - \frac{CI}{3\beta \sum_{j=1}^{N} \frac{1}{(1 + j)^N}} \tag{8}
\]

Using Eq. (8), the equilibrium quantities of production by different stakeholders can be determined for different values of \( \Delta \) and \( T \).

Two different policy maker preferences and the corresponding rational reaction sets of the stakeholders are shown in Figure 5. The two policy designers’ preferences are the two extreme conditions: a) maximization of overall quantity only without any cost considerations, and b) minimization of cost without any consideration to the quantity produced. The intersection of the rational reaction sets is the equilibrium quantities produced by two stakeholders.

![Figure 5 – Rational reaction sets for the two stakeholders under different preferences of policy maker.](image)

Figure 5 – Rational reaction sets for the two stakeholders under different preferences of policy maker.

Figure 6 displays the set of equilibria attained for different tradeoffs between policy design objectives and the corresponding values of policy design variables and the policy objectives. If the policy designer is only concerned with the cost of the policy (\( w_{11} = 0.0, w_{12} = 1.0 \)) the design variables will be selected to minimize this cost resulting in a low quantity of generation (each stakeholder will generate 298,854 KWh). The optimum values of decision variables in this scenario is \( \Delta = 0.0 \) $/KWh, \( T = 0.0 \) years, which results in low cost and low generation.

If these preferences are equal (\( w_{11} = 0.5, w_{12} = 0.5 \)) a moderate quantity will be generated (each stakeholder will generate 385,757 KWh). In this case, the optimum values of the decision variables are \( \Delta = 0.0 \) $/KWh, \( T = 21 \) years resulting in moderate quantity of generation yet a low cost of the policy. The policy will exist yet will not pay any premium-price yet will maximize the duration of the policy. In this case, the stakeholders will receive payments based on the market value only. No payment will be received in addition to the market value.

If the policy maker is only concerned with the quantity of energy being produced (\( w_{11} = 1.0, w_{12} = 0.0 \)), the production quantities will be chosen to maximize this value (in this case each stakeholder will generate 519,090 KWh). In this case, the optimum values of the variables are \( \Delta = 0.2 \) $/KWh, \( T = 21 \) years, which results in a high level of generation at a high policy cost.

6.2. Results for multiobjective policy design with incomplete preferences

In this section, we present the results of the policy design problem with incomplete preferences. The rational reaction sets for three preference scenarios for stakeholders are shown in Figure 7. For each case, the policy maker’s preference is fixed at \( w_{11} = 0.9 \) and the rational reaction sets are evaluated for the values of \( \Delta \) and \( T \) that maximize the policy maker’s payoff \( \pi \). The stakeholders’ preferences and corresponding equilibrium values of quantities are also shown in Figure 7. Since the policy maker’s preference is chosen to be greater for maximizing the quantity of generation (\( w_{11} = 0.9 \)), Scenario 2 will maximize this payoff with \( Q_{\text{tot}} = 944459 \) KWh. However, the policy maker does not have control over the preferences of the stakeholders and cannot guarantee what his/her payoff will be. This shows that for a fixed preference at the policy level, there is significant uncertainty in the equilibrium point if the knowledge about the preferences at the lower level is incomplete.

While the impact of incomplete preferences on the location of the Nash equilibrium is important, the important question from the policy maker’s standpoint is - what is the best decision for the policy maker to make? Although the preferences may not be completely known at the stakeholder level, the policy maker still has control over the decision variables. In Figure 8, we show the best decisions from the policy maker’s standpoint, and their impacts, under different preference scenarios. In Figure 8(a) it is assumed that both \( w_{211} \) and \( w_{212} \) are uncertain but are known to be within the range \([0.7, 0.8]\). For this uncertainty in stakeholder preferences, four preference conditions for the policy decision (\( w_{11} = 0.25, 0.75, 1.0 \)) are considered. It was also found that when \( w_{11} = 0.25, 0.39 \) or 0.5, the plots are identical. In Figure 8(b), it is assumed that uncertainty in \( w_{211} \) and \( w_{212} \) is lower and both weights are within the range \([0.79, 0.8]\). Although the regions where the equilibria may lie are still large, the corresponding policy design variables have either no uncertainty or significantly smaller uncertainty. For example, in Figure 8(a) the region corresponding to \( w_{11} = 1.0 \) has a large range of quantity values. Although this region is large, the policy maker’s best decision is to maximize both the premium-price and the duration of the policy. Therefore, despite the uncertainty in stakeholder preferences there is no uncertainty in design of the policy.

In Figure 8(c), the uncertainty in \( w_{211} \) and \( w_{212} \) is assumed to be between \([0.4, 0.5]\). Comparing this with Figure 8(a), it is observed that despite the same amount of uncertainty in the preferences, the uncertainty in the equilibrium values is significantly greater. In this case, the uncertainty in the choice of decision variables is also greater. This highlights the effects of the values of stakeholder preferences, in addition to the extent of uncertainty.
The results indicate that although there may be high uncertainty in the quantities generated at market equilibria, the policy maker may only have small uncertainty in the decision variables. This is primarily due to the decrease in uncertainty when the equilibria are mapped into the design space. We envision that in some policy design problems, the uncertainty may also increase when mapping the equilibria to design space. The results are sensitive to the market parameters ($\alpha, \beta$). By changing these parameters slightly we observed significant changes in the equilibrium quantities. Therefore, in a real FIT policy design, the market parameters need to be calibrated for the specific market under consideration. The results are more sensitive to the premium price and less sensitive to the duration of the policy. It was found that the premium-price of the policy motivated stakeholders to generate higher amounts of electricity. Although changing the duration of the policy had an effect, it was found that in most cases the duration of the policy was maximized. The complexity of the problem increases with the consideration of more stakeholders. Since we considered only two stakeholders, it was relatively easy to determine closed form solutions of equilibrium quantities. In order to consider multiple decision makers, techniques such as agent-based modeling may be employed.

7. CLOSING REMARKS

In this paper, we present a computational approach for policy design problems in sustainable energy systems such as the decentralized energy infrastructure. The specific focus is on policy design problems with multiple objectives and incomplete knowledge of preferences of the stakeholders. The lack of knowledge of the lower-level decision maker’s payoffs is one of the reasons for the failure of policies. Assumptions can be made as to what the objectives are and how these objectives are weighted but they may not be accurate. This inaccuracy can have detrimental effects creating an expensive non-successful FIT policy. This is typically the case as to why FIT policies fail [63]. Hence, there is a need to develop approaches to account for uncertainty resulting from the lack of complete information about stakeholder’s payoffs. In this paper, a specific class of incomplete preferences is addressed. It is assumed that the policy maker has knowledge about the different objectives of the stakeholders but has incomplete knowledge about how the stakeholders tradeoff different objectives.
The approach presented in this paper is based on multi-objective mathematical programs with equilibrium constraints (MOPECs), games with vector payoffs, and Nash equilibria of games with incomplete preferences. The primary contributions in this paper are mathematical formulation of the FIT policy, the extension of computational policy design problems to multiple objectives, and the consideration of incomplete preferences of stakeholders for policy design problems. The consideration of incomplete preferences is important for research on design under uncertainty because the existing design literature is primarily focused on uncertainty about physical phenomena but incomplete knowledge of preferences have received relatively lower attention.

While the motivation in this paper is that the policy designer has incomplete knowledge about the stakeholders’ payoffs, the approach can be used in two other situations also: a) the stakeholders may themselves not know what their preferences for tradeoffs are, b) the preferences of the stakeholders may represent group preferences. The second situation is common in many policy decisions because the problem is not only bilevel in nature – it is indeed multilevel in nature, as shown in the Figure 1. The proposed approach has
applications to other bilevel problems within engineering design research such as design for market systems [64, 65], fuel efficiency and emission policy [66, 67], and plug-in hybrid charging patterns [68]. All these problems are generally multi-objective in nature and require the knowledge of preferences of the stakeholders whose interactions result in market equilibria.

The proposed approach has limitations due to the assumptions made in this paper. First, it is assumed that the market behavior can be defined in terms of the Nash equilibrium. This is a common assumption made in the energy market modeling literature. However, in reality, the market is a dynamic system. Additionally, the decisions are not generally made by all stakeholders at the same time. The decisions may be made sequentially. Second, the approach presented in this paper is based on the assumption that the lower-level decisions can be converted into equilibrium constraints in the closed form. However, as the decisions of the stakeholders become more complex, deriving the equilibrium constraints in closed form may not be feasible. Finally, we do not consider stability of equilibria in this paper. The market equilibria for the problem presented in this paper happen to be stable for the ranges of decision variables considered. In a general case, the stability of the equilibria may change by changing the policy design variables. Price stability is an important aspect for the engineering design of distributed energy systems within smart electric grid. One of the goals of the policy design problem is to ensure the stability of the equilibrium. Integrating the stability considerations in the policy design problem is a challenge especially given the different possible stability problems such as price stability and voltage stability.

The illustrative example presented in this paper is also highly simplified. In the example, we consider only two energy producers whose quantity of generation is determined by the equilibrium. However, in practice, these energy producers are also required to meet local energy demands. The demand fluctuates with time, and the local producers can also purchase energy from central generation stations. The example presented in this paper is not based on specific RE technologies. One of the characteristics of these RE technologies is that their output is uncertain. In a holistic policy design framework, it is important to account for this uncertainty. The example is also based on the assumption that both stakeholders enter the market and make a decision at the same time. However, in practice, different stakeholders may enter the market at different times. Hence, the decisions are made at different time-steps with different amount of available information. These limitations clearly indicate the significant potential for further research in this direction.

8. ACKNOWLEDGEMENTS
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9. REFERENCES


