Module 16
Group Decision Making

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Decision Making in Engineering Design

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Module Outline

1. The Social Choice Problem
   - 1: Pairwise Majority
   - 2: Borda Aggregation Rule
2. Arrow's Impossibility Theorem
   - Theorem
   - Outline of Proof
   - Arrow's Theorem in Engineering Design
3. Aggregating Individuals' Preferences
   - 1. Aggregation under Certainty
   - 2. Aggregation Under uncertainty
   - Considerations of Equity

The Social Choice Problem

The Social Choice Problem

Social Choice Problem

Any decision problem faced by a group, in which each individual is willing to state at least ordinal preferences over the outcomes.

Example: Voice of the Customer

Question

What is your preferred choice of product color? A, B, C, D, or E

- A = Black
- B = Blue
- C = White
- D = Red
- E = Pink

Survey Results (100 customers):

- 45 prefer A
- 25 prefer B
- 17 prefer C
- 13 prefer D
- No one prefers E

Inference from the Survey: $A \succ B \succ C \succ D \succ E$

Preference Ordering

If designer considers **preference ordering** instead of just the top alternative, the following results are seen:

- 45 customers: $A \succ E \succ D \succ C \succ B$
- 25 customers: $B \succ E \succ D \succ C \succ A$
- 17 customers: $C \succ E \succ D \succ B \succ A$
- 13 customers: $D \succ E \succ C \succ B \succ A$

Verify that this preference structure will give the same results as seen on previous slide.

In fact, there are 120 such preference orderings that will give same survey results.

Preference Ordering (contd.)

1. Count the number of customers that prefer A to E... = ???
2. Count the number of customers that prefer E to A... = ???

What is the customers' **ACTUAL** preference?

Preference Ordering (contd.)

1. Number of customers that prefer A to E... = 45
2. Number of customers that prefer E to A... = 55

By performing similar pair-wise comparisons, it can be seen that the group's preference is: $E \succ D \succ C \succ B \succ A$, which is exactly opposite of the survey results!!!

It is also possible that there is no alternative preferred by the group.

Aggregation Rule

Aggregation Rule

A rule that transforms individual preferences (choices) into a social preference (choice).

Social Welfare Function

Any decision rule that aggregates a set of individual preference orderings over social states into a social preference ordering over those states.

1: Using the Pairwise Majority

Consider three individuals with three alternatives: A, B, & C

- Person 1's preference ordering: $A \succ B \succ C$
- Person 2's preference ordering: $B \succ C \succ A$
- Person 3's preference ordering: $C \succ A \succ B$

Condorcet's paradox (1785)!

Properties of the Majority Rule

- **Universal Domain:** The domain of admissible inputs of the aggregation rule consists of all logically possible profiles of votes.
- **Anonymity:** For any admissible voting profiles that are permutations of each other, the social decision is the same (all voters are treated equally).
- **Neutrality:** For any admissible profile, if the votes for the two alternatives are reversed, the social decision is reversed too (all alternatives are treated equally).
- **Positive Responsiveness:** For any admissible profile, if some voters change their votes in favour of one alternative, and all other votes remain the same, the social decision does not change in the opposite direction (social decision is a positive function of the way people vote).

Properties of the Majority Rule

Theorem by May (1952)

An aggregation rule satisfies universal domain, anonymity, neutrality, and positive responsiveness if and only if it is majority rule.

2: Borda Aggregation Rule

- Person 1's preference ordering: $A \succ B \succ C$
- Person 2's preference ordering: $B \succ C \succ A$
- Person 3's preference ordering: $C \succ A \succ B$

The Borda count avoids Condorcet's paradox but violates the **independence of irrelevant alternatives**.

Example: Material Selection

Material selection example: Consider material options A, B, C, D
The designer performs five experiments to rank order the materials based on the experiments. The experimental results are as follows:

- Test 1: $A \succ C \succ B \succ D$
- Test 2: $A \succ C \succ B \succ D$
- Test 3: $B \succ A \succ C \succ D$
- Test 4: $B \succ A \succ C \succ D$
- Test 5: $B \succ A \succ C \succ D$

Designers use a scoring method (Borda count) to select the material

- Winning material $\rightarrow$ 3 points
- Material at second place $\rightarrow$ 2 points and so on...
- Sum all the scores achieved by a material

Example: Material Selection (contd.)

- Test 1: $A \succ C \succ B \succ D$
- Test 2: $A \succ C \succ B \succ D$
- Test 3: $B \succ A \succ C \succ D$
- Test 4: $B \succ A \succ C \succ D$
- Test 5: $B \succ A \succ C \succ D$

Material A: $3 + 3 + 2 + 2 + 2 = 12$ points
Material B:
Material C:
Material D:

Example: Material Selection (contd.)

- Test 1: $A \succ C \succ B \succ D$
- Test 2: $A \succ C \succ B \succ D$
- Test 3: $B \succ A \succ C \succ D$
- Test 4: $B \succ A \succ C \succ D$
- Test 5: $B \succ A \succ C \succ D$

Material A: $3 + 3 + 2 + 2 + 2 =$ $\boxed{\text{12 points}}$
Material B: $1 + 1 + 3 + 3 + 3 = 11$ points
Material C: $2 + 2 + 1 + 1 + 1 = 7$ points
Material D: $0 + 0 + 0 + 0 + 0 = 0$ points

Decision

Select Material A.

Example: Material Selection (contd.)

Assume that the designer eliminates C from the selection

- Test 1: $A \succ B \succ D$
- Test 2: $A \succ B \succ D$
- Test 3: $B \succ A \succ D$
- Test 4: $B \succ A \succ D$
- Test 5: $B \succ A \succ D$

Material A: 3 + 3 + 2 + 2 + 2 = 12 points

Material B: 2 + 2 + 3 + 3 + 3 = 13 points

Material D: 0 + 0 + 0 + 0 + 0 = 0 points

Problem of irrelevant alternative!!!

The elimination of alternative C changed the designer's decision.

The problem can occur whenever the alternatives are used to establish the rating scale. This is a big problem! Designers seldom consider ALL POSSIBLE alternatives.

Other Aggregation Rules

1. Pairwise majority rule
2. Borda aggregation rule
3. Dictatorship
4. Weighted majority rule
5. Supermajority rules (Symmetrical and Asymmetrical)
6. Inverse majority rule

Properties of the Other Aggregation Rules

- Dictatorships and weighted majority rules with unequal individual weights violate **anonymity**.
- Asymmetrical supermajority rules violate **neutrality**.
- Symmetrical supermajority rules violate **positive responsiveness**.

Arrow's Impossibility Theorem

Arrow's Problem

Kenneth Arrow
1972 Nobel prize in Economics
2004 National Medal of Science

Arrow's Problem

Given the rankings of a set of alternatives by each individual in a decision making group, what should the group ranking of these alternatives be?

Arrow's Assumptions (1)

Assumptions

- *U: Unrestricted Domain*: There are at least two individuals in the group, at least three alternatives, and a group ordering is specified for all possible individual members' orderings (i.e., each individual is free to order the alternatives in any way).
- *P: (Pareto Principle) Positive Association of Social and Individual Orderings*:
  IF
  - the group ordering indicates $A \succ B$, and
  - individual's paired comparison between alternatives other than A are not changed, and
  - each individual's paired comparison between A and any other alternative either remains unchanged or is modified in A's favor

  THEN the group ordering must imply A is still preferred to B. (An ordinal version of monotonicity).
- *IIA: Independence of Irrelevant Alternatives*
- *S: Individual's Sovereignty*
- *ND: Non-dictatorship*

Arrow's Assumptions (2)

Assumptions

- *U: Unrestricted Domain*
- *P: (Pareto Principle) Positive Association of Social and Individual Orderings*
- *IIA: Independence of Irrelevant Alternatives*: If an alternative is eliminated from consideration and the preference relations for the remaining alternatives remain invariant for all the group members, then the new group ordering for the remaining alternatives should be identical to the original group ordering for these same alternatives. (Imposes transitivity in social order).
- *S: Individual's Sovereignty*: For each pair of alternatives A and B, there is some set of individual orderings such that the group prefers A to B.
- *ND: Non-dictatorship*: There is no individual with the property that whenever (s)he prefers alternative A to B, the group also prefers A to B regardless of the other individuals' preferences.

Arrow's Impossibility Theorem

Theorem (Arrow's Impossibility Theorem)

Assumptions U, P, IIA, S, and ND are inconsistent.

Interpretation: There is no procedure for combining individual rankings into a group ranking that does not explicitly address the question of **interpersonal comparison of preferences**!

Arrow's Impossibility Theorem: Proof Outline

Approach: Identify a "pivotal" voter, and establish that the pivotal voter is in fact a dictator.

Eric Pacuit: `https://www.youtube.com/watch?v=LVkioI5Z0JE`

Geanakoplos, J. (2004). *Three Brief Proofs of Arrow's Impossibility Theorem*, Cowles Foundation for Research in Economics. Discussion Paper No. 1123RRRR.

Arrow's Impossibility Theorem: Proof (1)

1. For any profile in which every agent puts alternative A at the very top or at the bottom, then the group must as well.

Arrow's Impossibility Theorem: Proof (2)

2. There exists a voter 'd' that by changing her vote at some profile, she can move A from the bottom of the group ranking to the top.

Arrow's Impossibility Theorem: Proof (3)

3. The agent 'd' is a dictator over any pair (B, C) not involving A.

Arrow's Impossibility Theorem: Proof (4)

The agent 'd' is also a dictator over every pair (A, C).

Arrow's Impossibility Theorem – Different versions

Theorem (Arrow's Impossibility Theorem – Version 1)

There exists no aggregation rule that satisfies the properties U, S, P, ND, and IIA.

Theorem (Arrow's Impossibility Theorem – Version 2)

If an aggregation rule satisfies the properties U, S, P and IIA, then it is dictatorial.

Arrow's view

Arrow's outlook is *individualistic* and *ordinalist*. It is not permitted to ask any further information to determine strength of preference.

According to Arrow, "interpersonal comparison of utilities has no meaning and ... there is no meaning relevant to welfare comparisons in the measurability of individual utility."

How relevant is Arrow's theorem for engineering design?

Analyzing Arrow's Assumptions for Engineering Design - I

"Arrow's impossibility theorem shows quite clearly that group utility functions do not, in general, exist. Therefore, any design methodology such as TQM or QFD that requires the construction and use of a group utility function is logically inconsistent and likely to produce erroneous results." (Hazelrigg, 1996)

Hazelrigg, G. A. (1996). "The Implication of Arrow's Impossibility Theorem on Approaches to Optimal Engineering Design", *ASME Journal of Mechanical Design*, Vol. 118, No. 2, pp. 161-164.

Analyzing Arrow's Assumptions for Engineering Design - II

"Arrow's Theorem does not apply to the multi-criteria engineering decision problem ... In engineering design, there may be many people involved, but decisions still depend upon the aggregation of engineering criteria" (Scott and Antonsson, 1999)

- In the social choice problem, all orderings are accorded equal worth. In the multiple criteria problem, it is desirable to be able to assign importance weightings to criteria.
- The social choice problem does not admit interpersonal comparison, while the multicriteria decision problem would be meaningless without inter-attribute comparisons.
- Engineering variables are almost always ordered on an external scale, and preferences for engineering requirements are commonly single-peaked around an ideal target.

Scott, M.J., Antonsson, E.K. (1999). "Arrow's Theorem and Engineering Design Decision Making", Research in Engineering Design, Vol. 11, pp.218-228.

Analyzing Arrow's Assumptions for Engineering Design - III

Arrow's impossibility theorem can affect engineering design in two ways: Aggregation of (a) preferences, and (b) design performances.

Impossibility theorem for performance aggregation:

- *Independence of irrelevant concepts*: The global performance structure between two design concepts depends only on their performances and not on other design concepts.
- *Non-dominance*: The global performance may not be determined by a single performance structure.
- *Unrestricted scope*: The performance structures on a single criterion as well as the global performance atructure are only restricted with respect to transitivity, reflexivity, and completeness.
- *Weak Pareto principle*: One design concept is strictly better than another concept in all the single performance structures $\Rightarrow$ the resulting global performance structure for these two concepts.

Jacobs, J.F., van de Poel, I., Osseweijer, P. (2014). "Clarifying the debate on selection methods for engineering: Arrow's impossibility theorem, design performances, and information basis", *Research in Engineering Design*, Vol. 25, pp. 3-10.

Arrow's Impossibility Theorem: Relevance to Engineering Design

Arrow's theorem carries over to the aggregation of other kinds of orderings, e.g.,

- belief orderings over several hypotheses (ordinal credences),
- multiple criteria that a **single decision maker** may use to generate an all-things-considered ordering of several decision options, and
- conflicting value rankings to be reconciled.

Relevance to Engineering Design:

- Aggregation of customers' preferences.
- Group decision making.
- Multi-criteria decision making (MCDM) methods.
- Evidence amalgamation in multi-scale modeling.
- ...

Aggregating Individuals' Preferences

The Group Decision Problem

Decision Maker (for the rest of this module): Person who wishes to incorporate the feelings, values, preferences, utilities of others into her own value assessments. The Decision Maker's preferences depend on the preferences of the others.

How can a (supra) Decision Maker systematically incorporate the views of others into her own decision making framework?

Keeney, R. L. and H. Raiffa (1993). *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. Cambridge, UK, Cambridge University Press. Chapter 10.

Aggregating Individuals' Preferences

From Ordinal to Cardinal Preferences

Assume that the consequences $\mathbf{x}$ of her decisions can be described in terms of attributes $X_1, X_2, \ldots, X_M$.

Overall objective: Maximize the well-being of N specified individuals.

N lower-level objectives: Maximize the well-being of individual i, defined by v_i or u_i.

1. The Certainty Model

(Supra) Decision Maker's value function: v over consequences $\mathbf{x}$

Individuals' value functions: $v_1, v_2, \ldots, v_N$

Goal: to find an appropriate model v_D such that

$$v(\mathbf{x}) = v_D[v_1(\mathbf{x}), v_2(\mathbf{x}), \ldots, v_N(\mathbf{x})]$$

1. The Certainty Model

Assumptions

Assumptions in this form:

1. Decision Maker's preferences are entirely captured by v_i's
2. Individual i's preference structure is completely captured by $v_i, \forall i$
3. Decision Maker knows all the v_i's

How realistic are these assumptions?

Complications

Situations where these assumptions may not be valid:

- The (supra) Decision Maker may have an interest in the outcomes, beyond the implications on individuals that she represents.
- The individuals themselves may not be capable of articulating their preferences to the Decision Maker.
- Individuals may not be honest.
- N may be large.

Restricting the Form of Group Value Functions

Assume that v_D exists, such that

$$v(\mathbf{x}) = v_D[v_1(\mathbf{x}), v_2(\mathbf{x}), \ldots, v_N(\mathbf{x})]$$

Assumption 1: Preferential independence

The attributes $\{V_i, V_j\}$ are preferentially independent of their complement $\bar{V}_{ij}$, for all $i \neq j, N \geq 3$.

Assumption 2: Ordinal positive association

Let certain alternatives A and B be equally preferred by the group. If A is modified to A' in such a manner that some individual i prefers A' to A but all other individuals remain indifferent, then A' is preferred to B by the group.

Assumption 1: Preferential Independence

Consider:

- a three-person group $N = 3$
- two consequences $\mathbf{x}'$ and $\mathbf{x}''$

Suppose the third individual is indifferent between $\mathbf{x}'$ and $\mathbf{x}''$.

The group preference of $\mathbf{x}'$ vs. $\mathbf{x}''$ should NOT depend on whether the third individual thinks $\mathbf{x}'$ and $\mathbf{x}''$ are awful or delightful. The supra Decision Maker's ranking should only depend on the feelings of individuals 1 and 2.

Assumption 2: Ordinal positive association

$$v(\mathbf{x}) = v_D[v_1(\mathbf{x}), v_2(\mathbf{x}), \ldots, v_N(\mathbf{x})]$$

Ordinal positive association: Let certain alternatives A and B be equally preferred by the group. If A is modified to A' in such a manner that some individual i prefers A' to A but all other individuals remain indifferent, then A' is preferred to B by the group.

Implication: v_D is a positive monotonic function of each of its arguments.

Additive Group Value Functions – Theorem

Theorem

Given $N \geq 3$, Assumption 1 (preferential independence) and Assumption 2 (ordinal positive association) hold if and only if

$$v(\boldsymbol{x}) = \sum_{i=1}^{N} v_i^*[v_i(\boldsymbol{x})] = \sum_{i=1}^{N} v_i^+(\boldsymbol{x})$$

where, for all i,

1. *v_i is a value function for individual i scaled from 0 to 1.*
2. *v_i^*, a positive monotonic transformation of its argument v_i, is the Decision Maker's value function over V_i, reflecting her interpersonal comparison of the individuals preferences.*
3. *v_i^+ defined as $v_i^*(v_i)$ is another value function for individual i consistently scaled to reflect the Decision Maker's interpersonal comparison of preference.*

2. The Uncertainty Model

Decision maker's utility function: u over consequences $\mathbf{x}$

Individuals' utility functions: $u_1(), u_2(), \ldots, u_N()$

Goal: to find an appropriate model such that

$$u(\mathbf{x}) = u_D[u_1(\mathbf{x}), u_2(\mathbf{x}), \ldots, u_N(\mathbf{x})]$$

Assumptions in this form:

1. Decision maker's preferences are entirely captured by u_i's
2. Individual i's preference structure is completely captured by $u_i, \forall i$
3. Decision maker knows all the u_i's

Example

Assume that the (supra) Decision Maker has a choice between:

- Certainty consequence C
- Lottery L: $\langle A, 0.5, B\rangle$

The Decision Maker ascertains that:

- For individual $i = 1$: $u_1^A = 1, u_1^B = 0, u_1^C = 0.4$
- For individual $i = 2$: $u_2^A = 0, u_2^B = 1, u_2^C = 0.4$

So, the Decision Maker has to choose between:

- C, which will result in utilities $(0.4, 0.4)$ for the two individuals
- L, which will result in $\langle(1, 0), 0.5, (0, 1)\rangle$

The decision maker must make **interpersonal comparisons**, i.e., the tradeoffs among the impacts on individuals $i = 1$ and $i = 2$.

Additive Group Utility Functions

Assume the existence of u_D such that

$$u(\mathbf{x}) = u_D[u_1(\mathbf{x}), u_2(\mathbf{x}), \ldots, u_N(\mathbf{x})]$$

In addition to this, the critical condition (Harsanyi, 1955) for

$$u(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i u_i(\mathbf{x})$$

is:

Assumption H (Harsanyi)

If two alternatives, defined by probability distributions over the consequences $\mathbf{x}$, are indifferent to each individual, then they are indifferent for the group as a whole.

Additive Independence and Strategic Equivalence

Assumption H is equivalent to the following pair of assumptions:

Assumption 3: Additive Independence

The set of attributes $U_1, U_2, \ldots, U_N$ is additive independent.

Assumption 4: Strategic Equivalence

The Decision Maker's conditional utility function u_i^* over the attribute U_i designating individual i's utility is strategically equivalent to individual i's utility function u_i. [Honesty Assumption]

Additive Independence and Strategic Equivalence

Theorem

For $N \geq 2$, Assumptions 3 (additive independence) and 4 (strategic equivalence) hold if and only if

$$u(\boldsymbol{x}) = \sum_{i=1}^{N} \lambda_i u_i(\boldsymbol{x})$$

where $u_i, i = 1, 2, \ldots, N$, is a utility function for individual i scaled from 0 to 1, the λ_i's are positive scaling constants, and $\boldsymbol{x}$ is a consequence.

The Decision Maker's interpersonal comparison of the individual's preferences is required to assess the λ_i scaling factors.

More General Group Utility Functions (1)

Assumption 5: Utility Independence

Attribute $U_i, i = 1, 2, \ldots, N$, is utility independent of the other attributes $\bar{U}_i$. This implies

$$u_D(u_1, \ldots, u_i, \ldots, u_N) = g_i(\bar{u}_i) + f_i(\bar{u}_i)u_i^*(u_i) \quad \forall i$$

Theorem

For $N \geq 2$, Assumptions 4 (strategic equivalence) and 5 (utility independence) imply

$$u(\boldsymbol{x}) = u_D(u_1, u_2, \ldots, u_N)$$

$$u(\boldsymbol{x}) = \sum_{i=1}^{N} \lambda_i u_i(\boldsymbol{x}) + \sum_{i=1, j>i}^{N} \lambda_{ij} u_i(\boldsymbol{x}) u_j(\boldsymbol{x}) + \cdots + \lambda_{12\ldots N} u_1(\boldsymbol{x}) u_2(\boldsymbol{x}) \ldots u_N(\boldsymbol{x})$$

More General Group Utility Functions (2)

Assumption 1A: Preferential Independence

The attributes $\{U_i, U_j\}$ are preferentially independent of their complement $\bar{U}_{ij}$ for all $i \neq j, N \geq 3$

Theorem

For $N \geq 3$, Assumptions 1 or 1A (preferential independence), 4 (strategic equivalence) and 5 (utility independence) imply

$$u(\boldsymbol{x}) = u_D(u_1, u_2, \ldots, u_N)$$

$$u(\boldsymbol{x}) = \sum_{i=1}^{N} \lambda_i u_i(\boldsymbol{x}) + \lambda \sum_{i=1, j>i}^{N} \lambda_i \lambda_j u_i(\boldsymbol{x}) u_j(\boldsymbol{x}) + \ldots$$
$$+ \lambda^{N-1} \lambda_1 \lambda_2 \ldots \lambda_N u_1(\boldsymbol{x}) u_2(\boldsymbol{x}) \ldots u_N(\boldsymbol{x})$$

where u and the u_i's are scaled from 0 to 1, the λ's are scaling constants, $0 < \lambda_i < 1$ for all i, and $\lambda > -1$.

Prior vs. Posterior Equity

A Decision Maker is interested in the preferences of two individuals and her utility function is:

$$u(\mathbf{x}) = u_D(u_1(\mathbf{x}), u_2(\mathbf{x})) = 0.5u_1(\mathbf{x}) + 0.5u_2(\mathbf{x})$$

Let $(0.4, 0.6)$ designate the alternative where $u_1 = 0.4$ and $u_2 = 0.6$

Consider the following alternatives:

- Alternative A: $(1, 0)$
- Alternative B: $\langle(1, 0), (0, 1)\rangle$
- Alternative C: $\langle(1, 1), (0, 0)\rangle$

Which of these alternatives should the Decision Maker choose?

Prior vs. Posterior Equity

The additive utility function cannot differentiate among Alternatives *A*, *B*, *C*.

Alternatives *B* and *C* are *a priori* equitable, whereas alternative *A* is not.

Alternative *C* is more equitable in terms of *posterior* equity.

Prior vs. Posterior Equity

Another utility function:

$$u(\mathbf{x}) = 0.4u_1(\mathbf{x}) + 0.4u_2(\mathbf{x}) + 0.2u_1(\mathbf{x})u_2(\mathbf{x})$$

Expected utilities for alternatives A, B, and C are $0.4, 0.4$, and 0.5 respectively.

Provides a formal consideration of *posterior equity*.

Pareto Optimality

Consider the following alternatives:

- Alternative A: $(1, 0)$
- Alternative B: $\langle(1, 0), (0, 1)\rangle$
- Alternative C: $\langle(1, 1), (0, 0)\rangle$
- Alternative D: $(0.48, 0.48)$

If $u(\mathbf{x}) = 0.5u_1(\mathbf{x}) + 0.5u_2(\mathbf{x})$, then $B \succ D$
If $u(\mathbf{x}) = 0.4u_1(\mathbf{x}) + 0.4u_2(\mathbf{x}) + 0.2u_1(\mathbf{x})u_2(\mathbf{x})$, then $D \succ B$

The decision maker is faced with a tradeoff of amount of equity versus degree to which the narrow interpretation of Pareto optimality is violated.

Universal Agreement

Assumption: Universal Agreement

If all members of a group have the same utility function, then the group utility function should be the common utility function.

In some cases, this assumption may not be reasonable! For example, the group may be less risk averse than the individuals.

Can you think of a reason?

Assumption 5: Utility Independence

Utility independence may not be appropriate in certain conditions.

Consider two situations where all but one individual (*i*) is indifferent between the consequences:

1. All individuals that are indifferent between alternatives are impacted in a very **desirable** manner.
2. All individuals that are indifferent between alternatives are impacted in a very **undesirable** manner.

If everyone's utility is low, then because of the pressures among individuals, the Decision Maker may prefer consequences where the individual (*i*) also receives an undesirable consequence rather than a desirable one.

Other Decision Problems (NOT covered in this Module)

Interactive Decisions

When the decisions made by one decision maker depend on the decisions made by other decision makers.

Summary: Challenges in Social Decision Making

Challenges

- Conflicts of interest. Different people may prefer different things.
- Individuals will want to follow different choice functions.
- The aggregation rule should preserve the desirable properties of individual preferences.
- There are additional desirable properties for the social context.
- Individuals may behave strategically.

Summary

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   - 1: Pairwise Majority
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   - Theorem
   - Outline of Proof
   - Arrow's Theorem in Engineering Design
3. Aggregating Individuals' Preferences
   - 1. Aggregation under Certainty
   - 2. Aggregation Under uncertainty
   - Considerations of Equity

References – Social Choice Theory

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