Module 11
Rationality

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Decision Making in Engineering Design

PURDUE MECHANICAL ENGINEERING

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Module Outline

Motivation

Behavioral Axioms

- **Ordering of alternatives**: Decision maker can order any two alternatives and the ordering is transitive.
  Transitivity: If $A_1 \succ A_2, A_2 \succ A_3$, then $A_1 \succ A_3$
- **Reduction of compound uncertain events** using complicated mixtures of lotteries using probability theory.

$$\langle x^*, 0.2, x_0 \rangle \sim \boxed{???}$$

$$\langle \langle x_1, 0.2, x_2 \rangle, 0.5, \langle x_3, 0.5, x_4 \rangle \rangle \sim \langle \boxed{???}, 0.2, x_0 \rangle$$

$$\langle \langle (x_1, y_1), 0.2, (x_2, y_2) \rangle, 0.5, \langle (x_3, y_3), 0.5, (x_4, y_4) \rangle \rangle \sim \langle \boxed{(???, ???)}, 0.2, (x_0, y_0) \rangle$$

- **Substitutability**: reverse of reduction (i.e., the decision maker is willing to substitute a deterministic outcome with a lottery).

Clemen, R. T. (1996). *Making Hard Decisions: An Introduction to Decision Analysis*. Belmont, CA, Wadsworth Publishing Company. Chapter 14.

Behavioral Axioms (contd.)

- **Continuity**: If $A_1 \succ A \succ A_2$, then we can construct a lottery between A_1 and A_2 with probability $0 \leq p \leq 1$ so that the decision maker is indifferent between A and the lottery.
- **Monotonicity**: Decision maker prefers the gamble which has higher probability of achieving the preferred outcome.
- **Invariance**: All that is needed to determine a decision maker's preferences among uncertain events are the payoffs and the associated probabilities.
- **Finiteness**: No consequences are infinitely good or infinitely bad.

Clemen, R. T. (1996). *Making Hard Decisions: An Introduction to Decision Analysis*. Belmont, CA, Wadsworth Publishing Company. Chapter 14.

Questions for this Module

Questions for the Day

Do people really satisfy these behavioral axioms?
Are people rational (as defined)?

Allais Paradox – Decision 1

Decision 1: Choose A or B

A: Win \$1 million with probability 1

B: Win \$5 million with probability 0.10
Win \$1 million with probability 0.89
Win \$0 million with probability 0.01

Allais Paradox – Decision 2

Decision 2: Choose C or D

C: Win $1 million with probability 0.11
Win $0 with probability 0.89

D: Win $5 million with probability 0.10
Win $0 million with probability 0.90

Allais Paradox – Experimental Results

Experiments show that as many as 82% of the subjects prefer A over B, and 83% prefer D over C.

This violates the expected utility maximization principle. Why?

Allais Paradox – Expected Utility Calculations

Decision 1

Decision 1: Choose A or B

✓**A:** Win \$1 million with probability 1

B: Win \$5 million with probability 0.10
Win \$1 million with probability 0.89
Win \$0 million with probability 0.01

Let $U(\$0) = 0$ and $U(\$5M) = 1$

EU(A) = U(\$1 Million)
EU(B) = $0.10\ (1) + 0.89 \times U(\$1\text{ million}) + 0.01\ (0)$
So, A is preferred to B if and only if $U(\$1\text{ million}) > 0.91$

Allais Paradox – Expected Utility Calculations

Decision 2

Decision 2: Choose C or D

C: Win $1 million with probability 0.11
Win $0 with probability 0.89

✓**D:** Win $5 million with probability 0.10
Win $0 million with probability 0.90

EU(C) = 0.11 $\times$ U($1 million)
EU(D) = 0.10
So, D is preferred to C if and only if U($1 million) $<$ 0.91

Tversky and Kahneman

Tversky and Kahneman

Kahneman: 2002 Nobel Memorial Prize in Economic Sciences

"People reply on a limited number of heuristic principles which reduce the complex task of assigning probabilities and predicting values to simpler judgmental operations."

Examples of heuristics that people use in assessing probabilities:

1. Representativeness
2. Availability
3. Adjustment and anchoring

The use of these heuristics can lead to **systematic** errors!

Tversky, A. and Kahneman, D., 1974, "Judgment under Uncertainty: Heuristics and Biases," Science, 185(4157), pp. 1124-1131.

Representativeness

1. What is the probability that object A belongs to class B?
2. What is the probability that event A originates from process B?
3. What is the probability that process B will generate event A?

Probabilities are evaluated by the degree to which A is representative of B (i.e., the degree to which A resembles B).

Representativeness:
a) Estimating Probability from Similarity

"Steve is a very shy and withdrawn, invariably helpful, but with little interest in people, or in the world of reality. A meek and tidy soul, he has a need for order and structure, and a passion for detail."

Assess the probability that Steve is engaged in a particular occupation:

- farmer
- salesman
- airline pilot
- librarian
- physician

Representativeness: b) Insensitivity to prior probability of outcomes

Consider a school in a research university with 70 graduate students and 30 faculty members.

> John is a 30 year old man. He is married with no children. A man of high ability and high motivation, he promises to be quite successful in his field. He is well liked by his colleagues.

Assess the probability that John is a faculty member.

Representativeness: b) Insensitivity to prior probability of outcomes

When no specific evidence is given, prior probabilities are poorly utilized; when worthless evidence is given, prior probabilities are ignored.

Representativeness: c) Insensitivity to sample size

A certain town is served by two hospitals. In the larger hospital about 45 babies are born each day, and in the smaller hospital, about 15 babies are born each day. As you know, about 50% of all babies are boys. However, the exact percentage varies from day to day. Sometimes it may be higher than 50%, sometimes lower.

For a period of 1 year, each hospital recorded the days on which more than 60 percent of the babies born were boys. Which hospital do you think recorded more such days?

1. The larger hospital
2. The smaller hospital
3. About the same (i.e., within 5% of each other)

Representativeness: d) Misconceptions of chance

In a sequential coin toss, which sequence is more likely?

"H-T-H-T-T-H"
or
"H-H-H-T-T-T"
or
"H-H-H-H-T-H"

Expectation that a sequence of events generated by a random process will represent the essential characteristics of that process even when the sequence is short.

Representativeness: d) Misconceptions of chance

Gambler's Fallacy: Chance is viewed as a self-correcting process in which deviations in one direction induces a deviation in the opposite direction to restore the equilibrium.

In reality, deviations are not "corrected", they are merely diluted.

Representativeness: e) Insensitivity to predictability

Predict the future profit of the following two companies:

1. **Beyond Meat**: “We believe there is a better way to feed the planet. Our mission is to create mass-market solutions that perfectly replace animal protein with plant protein. We are dedicated to improving human health, positively impacting climate change, conserving natural resources and respecting animal welfare. At Beyond Meat, we want to make the world a better place and we’re starting one delicious meal at a time.”
2. **Leap Motion:** Leap Motion, Inc. is an American company that manufactures and markets a computer hardware sensor device that supports hand and finger motions as input, analogous to a mouse, but requiring no hand contact or touching.
   News (March 20, 2014): Leap Motion Lays Off 10% Of Its Workforce After Missing On First Year Sales Estimates.

Representativeness: f) Misconceptions of regression

Flight training example – Experienced flight instructors noted that praise for an extremely smooth landing is typically followed by a poor landing on the next try, while a harsh criticism after a rough landing is usually followed by an improvement on the next try.

Conclusion: verbal rewards are detrimental to learning, while verbal punishments are beneficial.

Is this a correct conclusion? Why? Why not?

Representativeness: g) Misconceptions of regression (contd.)

Regression towards the mean: Consider two variables X and Y which have the same distribution. If one selects individuals whose average X score deviates from the mean by k units, then the average of their Y scores will usually deviate from the mean of Y by less than k units.

Example: Suppose a large group of children has been examined on two equivalent versions of an aptitude test. If one selects ten children from among those who did best on one of the two versions, he/she will usually find their performance on the second version to be somewhat disappointing.

2. Availability

Situations in which people assess the frequency of a class or the probability of an event by the ease with which instances or occurrences can be brought to mind.

Instances of large classes are usually recalled better and faster than instances of less frequent classes.

Availability: a) Biases due to retrievability of instances

Experiment: Subjects heard a list of well-known personalities of both sexes and were asked to judge whether the list contained more names of men than women.

Outcome: In each list, the subjects erroneously judged that the gender that had the most famous personalities was the more numerous.

Other related factors that affect retrievability:

1. recent vs. distant past
2. seeing vs. reading about it

Availability:
b) Biases due to the effectiveness of a search set

Are there more words in the English language that start with ‘r’ than words with ‘r’ as the third letter?

Availability:
c) Biases of imaginability

Consider a group of 10 people who form committees of k members, $2 \leq k \leq 8$.

How many different committees of k members can be formed?

3. Adjustment and Anchoring

In many situations, people make estimates by starting from an initial value that is adjusted to yield the final answer.

Different starting points yield different estimates, which are biased toward the initial values.

Adjustment and anchoring: a) Insufficient adjustment

1. Subjects asked to estimate various quantities stated in percentages (e.g., percentage of African countries in the United Nations).
2. A number between 0 and 100 is determined by spinning a wheel of fortune.
3. The subjects were instructed to indicate whether that number was higher or lower than the value of the quantity, and then to estimate the value of the quantity by moving upward or downward from the given number.

Results – Different groups were given different numbers for each quantity, and these arbitrary numbers had a marked effect on the estimates.

Adjustment and anchoring: b) Biases in the evaluation of conjunctive and disjunctive events

Subjects given opportunity to bet on one of two events.

Three types of events:

1. *Simple events*: drawing a red marble from a bag containing 50% red marbles and 50% white marbles.
2. *Conjunctive events*: drawing a red marble seven times in succession, with replacement, from a bag containing 90% red marbles and 10% white marbles.
3. *Disjunctive events*: drawing a red marble at least one in seven successive trials, with replacement, from a bag containing 10% red marbles and 90% white marbles.

Adjustment and anchoring: b) Biases in the evaluation of conjunctive and disjunctive events

Conclusion: People tend to overestimate the probability of conjunctive events and to underestimate the probability of disjunctive events.

The stated probability of the elementary event provide a natural starting point for estimation. The final estimates remain too close to the starting points.

Caution: Because of anchoring, people will tend to underestimate the probabilities of failure in complex systems. The chain like structure of conjunctions lead to overestimation, the funnel like structure of disjunctions leads to under-estimation.

Simon – Bounded Rationality

Simon's Argument

"Because of the psychological limits of the organism (particularly with respect to computational and predictive ability), actual human rationality-striving can best be an extremely crude and simplified approximation to the kind of global rationality that is implied."

Need to "replace the global rationality of economic man with a kind of rational behavior that is compatible with the access to information and the computational capacities that are actually possessed by organisms, including man, in the kinds of environments in which such organisms exist."

H. A. Simon, "A Behavioral Model of Rational Choice," *The Quarterly Journal of Economics*, Vol. 69, No. 1 (Feb., 1955), pp. 99-118.

Simon's Simplifications

Severe demands placed by utility theory.

1. Ability to specify the exact nature of the outcomes.
2. Payoffs must be completely ordered.
3. Either the outcomes of alternatives must be known with certainty, or it must be possible to attach definite probabilities to the outcomes.

Simon proposed three deviations in his behavioral model of rational choice:

1. Simple payoff functions.
2. Information gathering.
3. Partial ordering of payoffs.

a) Simple Payoff Functions

Assume that the value function $V(s)$ assumes only two values, $(1, 0)$, interpreted as satisfactory or unsatisfactory.

Example

S represents possible prices for a house an individual is selling. He may regard $100,000 as "acceptable", anything over this amount as "satisfactory" and anything below that as "unsatisfactory".

The $100,000 amount would be referred to as the aspiration level.

b) Information Gathering

1. In some cases, the value function may be known in advance, but the mapping of alternatives onto the performance space may not be known.
2. Gathering information is not always costless.
3. Narrowing down the search space to reduce the cost of mapping the alternatives to the performance.
4. Role of the design process.

c) Partial Ordering of Payoffs

"The classical utility theory does not tolerate the incomparability of oranges and apples."

Instead of a scalar payoff function, $V(s)$, we might have a vector function, $V(s)$; where V might have components $V_1, V_2, \ldots, V_n$

This is particularly true if:

1. decisions are made by *groups of individuals*. The components may represent the payoff functions of the individual members of a group.
2. an individual is trying to implement a number of values that do not have a common denominator
3. where each behavior alternative maps to n possible consequences.

Deviations from Expected Utility Theory

Linearity in Probabilities

Expected Utility theory imposes linearity in probabilities, i.e.,

$$E(U) = \sum_i p_i U(x_i)$$

Example: For a lottery: $\langle x^*, p, x_0 \rangle$,

$$E(U) = pU(x^*) + (1 - p)U(x_0)$$

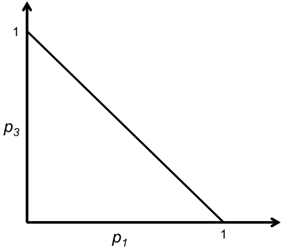
If $x^* > x_0$, increasing p results in stochasically dominating lotteries, i.e.,

$$\langle x^*, p + \Delta, x_0 \rangle \succ \langle x^*, p, x_0 \rangle$$

Linearity in Probabilities

Consider a set of all lotteries over the fixed outcome levels $x_1 < x_2 < x_3$, represented by the probability triples $P = (p_1, p_2, p_3)$ such that $\sum_i p_i = 1$. Therefore,

$$p_2 = 1 - (p_1 + p_3)$$



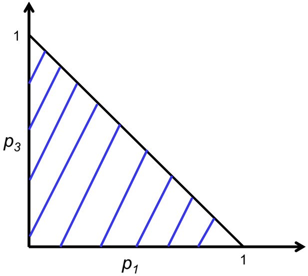
Machina, M.J., 1987, "Choice Under Uncertainty: Problems Solved and Unsolved," *Economic Perspectives*, Vol. 1, No. 1, pp. 121-154.

Indifference Curves

Indifference curves:

$$\langle (x_1, p_1), (x_2, p_2), (x_3, p_3) \rangle \sim \langle (x_1, p_1'), (x_2, p_2'), (x_3, p_3') \rangle$$

Alternatively,
$E(U) = \sum_{i=1}^{3} U(x_i)p_i = p_1 U(x_1) + (1 - p_1 - p_3) U(x_2) + p_3 U(x_3) =$ constant



Slope of the indifference curves:

$$\frac{U(x_2) - U(x_1)}{U(x_3) - U(x_2)}$$

Behaviorally, the property of linearity in probabilities can be viewed as a restriction on the individual's preferences over probability mixtures of lotteries.

The Independence Axiom

The Independence Axiom

If $X \succ Y$, then

$$pX + (1-p)Z \succ pY + (1-p)Z \qquad \forall Z, p \in (0,1)$$

Independence implies indifference curves are parallel straight lines.

Violations of Linearity in the Probabilities

Allais Paradox

Decision 1: Choose A or B

✓**A:** Win $1 million with probability 1

B: Win $5 million with probability 0.10
Win $1 million with probability 0.89
Win $0 million with probability 0.01

Decision 2: Choose C or D

C: Win $1 million with probability 0.11
Win $0 with probability 0.89

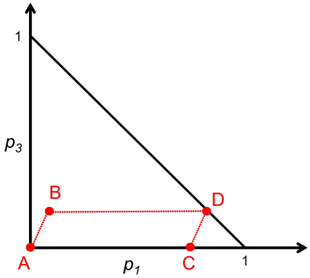
✓**D:** Win $5 million with probability 0.10
Win $0 million with probability 0.90

Violations of Linearity in the Probabilities

Allais Paradox

Define
$\{x_1, x_2, x_3\} = \{\$0, \$1M, \$5M\}$

- **Option A**:
  $p_1 = 0, p_2 = 1, p_3 = 0$
- **Option B**:
  $p_1 = 0.01, p_2 = 0.89, p_3 = 0.1$
- **Option C**:
  $p_1 = 0.89, p_2 = 0.11, p_3 = 0$
- **Option D**:
  $p_1 = 0.9, p_2 = 0, p_3 = 0.1$



The Common Ratio Effect

Decision 1: Choose A or B

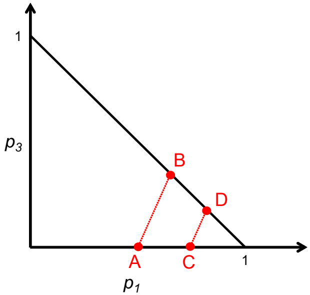
✓**A:** Win X with probability p
Win \$0 with probability $(1 - p)$

B: Win Y with probability q
Win \$0 with probability $(1 - q)$

Decision 2: Choose C or D

C: Win X with probability rp
Win \$0 with probability $(1 - rp)$

✓**D:** Win Y with probability rq
Win \$0 with probability $(1 - rq)$



Generalizations of Expected Utility Theory

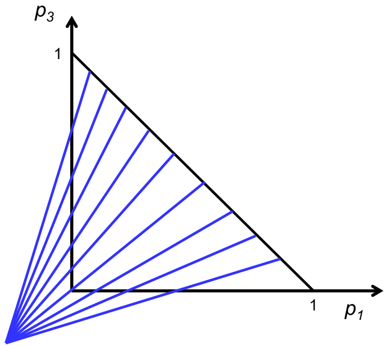
Generalizations of Expected Utility to accommodate these observations.

- Weighted Utility Theory
- Implicit Expected Utility Theory
- Expected Utility with Rank-Dependent Probability Weights
- Lottery Dependent Utility Theory
- Prospect Theory (to be discussed in detail in the next Module)

a. Weighted Utility Theory

$$U(\sum_i p_i x_i) = \frac{\sum_i p_i w(x_i) u(x_i)}{\sum_i p_i w(x_i)}$$

Resulting indifference curves:



b. Implicit Expected Utility Theory

Implicit Utility Representation:

$$U^* = \sum_i p_i u(x_i, U^*)$$

Indifference curves are straight lines, but not parallel.

c. Expected Utility with Rank-Dependent Probability Weights

The weight of an outcome depends on is probability **and its rank order** in the set of possible outcomes.

Say $x_1 > x_2 > x_3 > \ldots > x_n$. Then, the rank dependent utility is:

$$U = \sum_{i=1}^{n} u(x_i) \underbrace{\left[g(p_1 + p_2 + \ldots + p_{i-1} + p_i) - g(p_1 + p_2 + \ldots + p_{i-1})\right]}_{\text{Weight of } x_i}$$

Here, the indifference curves are not straight lines, unless $g(p) = p$.

d. Lottery Dependent Utility Theory

$$U = \sum p_i u(x_i, c_F)$$

where

$$c_F = \sum h(x_i) p_i$$

The indifference curves are non-linear. They fan-out in exponential form.

Other Deviations from Expected Utility Theory

- Ambiguity aversion
- Preference Reversals – Framing Effects
- Preference Reversals – Violations of Procedure Invariance

The Ellsberg Paradox

There are 30 Red balls and 60 balls in some combination of Black and Yellow.

Choice	Red	Black	Yellow
A	$1000	$0	$0
B	$0	$1000	$0
C	$1000	$0	$1000
D	$0	$1000	$1000

Decision 1: Choose between A and B
Decision 2: Choose between C and D

Ambiguity Aversion

Ambiguity: "Known to be missing information".

Finding from experiments: People prefer acts with known probabilities of winning.

Ambiguity aversion implies there may be a gap between subjects' beliefs about an event's likelihood and their willingness to bet on the event. This is a deviation from Subjective Expected Utility theory.

Implications: People who know a lot about a domain of events prefer betting on events.

Preference Reversals

Framing Effects

Problem 1

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people.
Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

- If Program A is adopted, 200 people will be saved. [72 percent]
- If Program B is adopted, there is 1/3 probability that 600 people will be saved, and 2/3 probability that no people will be saved. [28 percent]

Which of the two programs would you favor?

Preference Reversals

Framing Effects

Problem 2

Imagine that the U.S. is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people.
Two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs are as follows:

- If Program C is adopted 400 people will die. [22 percent]
- If Program D is adopted there is 1/3 probability that nobody will die, and 2/3 probability that 600 people will die. [78 percent]

Which of the two programs would you favor?

Preference Reversals

Equivalent Procedures for Estimating Utility Functions

1. Preference comparison methods: $\langle x', p, x'' \rangle$??? x
2. Probability equivalence methods: $\langle x_{i+1}, ???, x_{i-1} \rangle \sim x_i$
3. Value equivalence methods: $\langle x_{i+1}, 0.5, x_{i-1} \rangle \sim x_i$
4. Certainty equivalence methods $\langle x_n, p, x_0 \rangle \sim x_i$
5. Preference comparison methods: $\langle x', p, x'' \rangle$??? $\langle x''', p, x'''' \rangle$
6. Probability equivalence methods: $\langle x_{i+1}, 0.5, x_{i-1} \rangle \sim \langle x_{n+1}, ???, x_0 \rangle$
7. Value equivalence methods: $\langle ???, p_1, x'' \rangle \sim \langle x''', p_2, x'''' \rangle$

Preference Reversals

Violations of Procedure Invariance

Ideally, the preference relations should not be affected by different elicitation methods.

Examples of two mathematically equivalent methods:

1. Choosing between two lotteries (preference comparison method)
2. Determining certainty equivalents of the lotteries (certainty equivalence methods)

Consider two bets:

- **P-bet**: High probability and low payoff
- **$-bet**: Low probability and high payoff

Subjects chose the P-bet using the preference comparison method, but valued the $-bet higher based on the certainty equivalence method.

Cognitive Psychology Perspective

Humans as information processors, employing processes of perception, memory, categorization, problem solving and so on.

1. Capacity Limitations: Limits on how much information can be processed
2. Interactions between attention and memory: Working memory
3. Distinction between Controlled and Automatic Processing
4. Ability to learn: Adaptation of behavior
5. Categorization: Process of organizing knowledge
6. Meta-cognition: Thinking about own thinking (cost of cognition)

Newell, B. R., Bröder, A., 2008, Cognitive processes, models and metaphors in decision research, *Judgment and Decision Making*, Vol. 3, No. 3, pp. 195-204.

a. Capacity Limitations

Limitations imposed by:

- **Cognitive capacity**: The limit on the amount of information that an organism can attend to and/or process at any given time (e.g., attention span, memory).
- **Environment**: Environment shapes the human behavior (e.g., information costs).

Addressing cognitive and environmental limitations

Simplifying heuristics help in making good enough ("satisficing") decisions. Collections of heuristics for specific tasks.

Heuristics to Deal with Capacity Limitations

1. **Recognition heuristic**: When choosing between two objects, if one is recognized and the other is not, then select the former. When deciding which companies to invest in from among those trading in a particular stock market, the recognition heuristic would lead investors to choose just those that they have heard of before.
2. **One-reason decision making**: Stop looking for cues as soon as one is found that differentiates between the two options being considered.
3. **Elimination heuristics** for multiple-option choices: Successive cues are used to eliminate more and more alternatives and thereby reduce the set of remaining options, until a single option can be decided upon.
4. **Satisficing heuristics**: An aspiration level is set for the selection criterion being used, and the search for alternatives is stopped as soon as the aspiration is met

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Gigerenzer, G., Todd, P.M., 1999, *Simple heuristics that make us smart*, Oxford University Press.

b. Automatic vs. Controlled Processing

Controlled processing

Information processing takes place in a controlled, serial manner.

Example: Elimination heuristics.

Automatic processing

Information processing happens automatically and in parallel (e.g., vision).

Individuals encounter a decision situation, salient and associated information is activated in memory and a mental representation is formed that combines given and memory stored information: active network.

c. Learning

Judgments and decisions are informed by prior experience. Each decision yields some information that can add to our stock of experience for future benefit.

Over time, we can learn to adaptively alter our behavior to improve our decision making.

d. Categorization

Categorization is a fundamental ability which allows us to organize our knowledge, react appropriately and make useful predictions about the properties of things we encounter in the world.

Two models:

- *Exemplar models*: reliance on specific instances of events or objects retrieved from memory.
- *Controlled cue abstraction*: integration of cue-criterion values that had been abstracted via training or experience in relevant environments.

Descriptive, Normative, and Prescriptive Models

1. **Descriptive Models:** Accurate specifications of the response patterns of human beings and theoretical accounts of those observed patterns in terms of psychological mechanisms.
2. **Normative Models:** Embody standards of cognitive activity - standards that, if met, serve to optimize the accuracy of beliefs and the efficacy of actions. (e.g., expected utility theory).
3. **Prescriptive Models:** specify how processes of belief formation and decision making should be carried out, given the limitations of the human cognitive apparatus and the situational constraints (e.g., time pressure) with which the decision maker must deal.

Stanovich, K.E. (1999) *Who is Rational? Studies of Individual Differences in Reasoning*, Psychology Press, Mahwah, NJ.

Three positions on human rationality

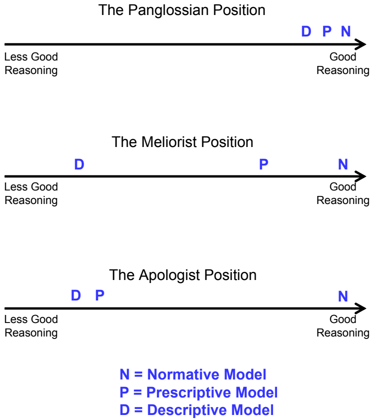

Figure: 1.1 on page 5 (Stanovich)

Alternative Explanations of Normative – Descriptive Gap

1. Systematic irrationality in the intentional-level psychology
2. Algorithmic-level limitations
3. Performance errors
4. Application of inappropriate normative model
5. Alternative problem construal

Summary

1. Motivation
   - Behavioral Axioms
   - Allais Paradox
2. Tversky and Kahneman
   - 1. Representativeness
   - 2. Availability
   - 3. Adjustment and Anchoring
3. Simon – Bounded Rationality
   - Simon's Argument
   - Simon's Behavioral Model of Rational Choice
4. Deviations from Expected Utility Theory
   - Generalizations of Expected Utility Theory
   - Other Deviations from Expected Utility Theory
   - Cognitive Psychology Perspective
   - Descriptive, Normative and Prescriptive Models

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