Module 09
Value of Information

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Decision Making in Engineering Design

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Module Outline

Clemen, R. T. (1996). Making Hard Decisions: An Introduction to Decision Analysis. Belmont, CA, Wadsworth Publishing Company. Chapter 12.

Howard, R.A. and Abbas, A.E. (2016). Foundations of Decision Analysis. Person, London. Chapter 13.

Introduction to Value of Information

Sources of Information in Engineering Design

Information Sources:

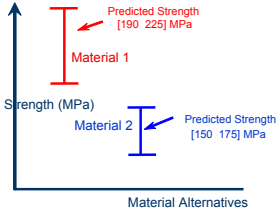
- Consultants
- Simulation Models
- Experiments
- ...

Information is costly!

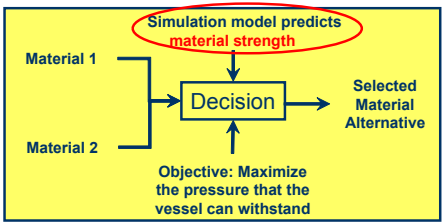
Should you acquire information (or source of information)?
If so, how much should you pay for it?

Is Further Refinement Necessary? A Conceptual Example

How much refinement of a simulation model is appropriate for design?



Should this model be refined further?

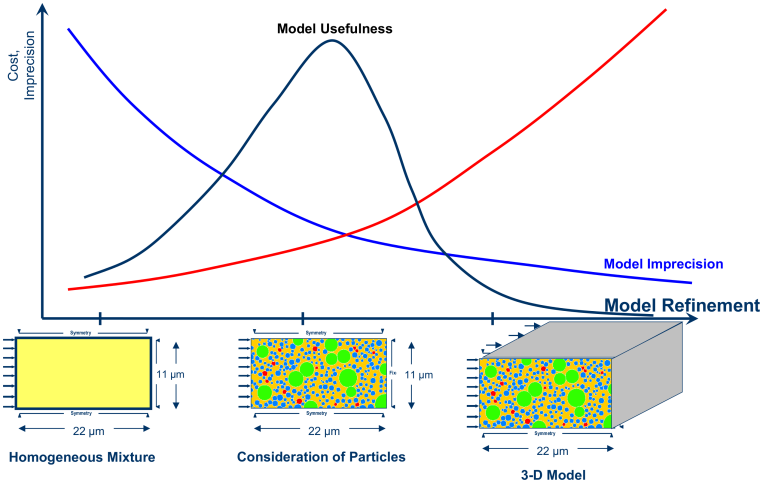


Inferences

- Improved accuracy does not imply improved usefulness in design !!!
- The appropriateness depends on the overall design problem (constraints, objectives, and variable bounds)

Model Refinement Using Value-of-Information

All models are wrong, some models are useful! [George Box]



Basic Questions

Questions to be addressed:

- What is an appropriate basis on which to evaluate the **value of information** in a decision situation?
- What does it mean for an expert to provide **perfect information**?
- How does probability relate to the idea of information?

Illustrative Example (1)

Buying a Rain Predictor

Three alternatives:

- Party outdoors
- Party on porch
- Party indoors

Uncertainty: Payoff depends on whether it is sunny or it rains.

Information acquisition decision: Whether to purchase a rain predicting sensor.

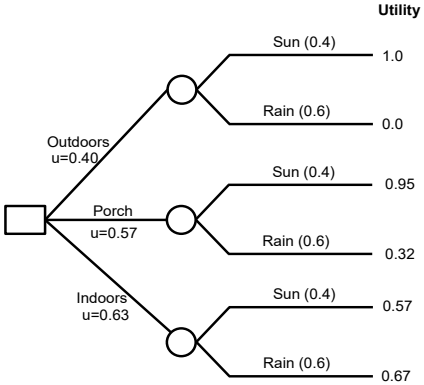


Figure: 10.7 on Page 201 (Howard and Abbas)

Illustrative Example (2)

Investing in the Stock Market

Investor has three alternatives:

- High-risk stock
- Low-risk stock
- Savings account

Uncertainty: Payoff on stocks depends on whether the market goes up, remains same, or goes down.

Information acquisition decision: Whether to get expert advice.

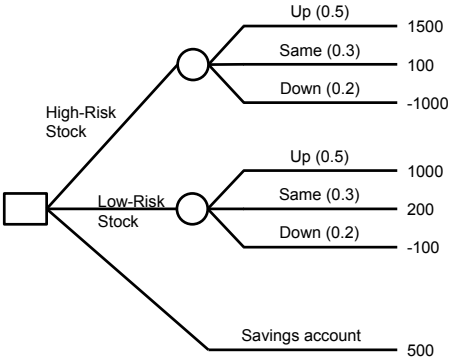


Figure: 12.1 on Page 436 (Clemen)

The Expected Value of Information – Key Concepts (1)

If the decision maker will make the same decision regardless of what the new information is, then the information has no value!

Rain Prediction Sensor Example:

- The best decision without the sensor is to hold the party indoors.
- If after buying the sensor, the decision still remains the same (irrespective of what the sensor shows) $\Rightarrow$ no value.

The Expected Value of Information – Key Concepts (2)

Value of information needs to be determined before actually getting the information (e.g., before hiring the expert).

- *Worst case scenario*: Decision remains the same even after hiring the expert $\Rightarrow$ Zero value of information.
- *Better scenarios*: Expected value increases $\Rightarrow$ Positive value of information.
- *Best case scenario*: Perfect Information (resolving all uncertainty; Expert tells us exactly what will happen) $\Rightarrow$ Maximum value of information (i.e., Expected Value of Perfect Information).

Expected Value of Perfect Information

The Notion of "Perfect Information"

Clairvoyant: An expert's information is said to be perfect if it is **always correct**.

1. When state S will occur, the information source always says so.
   - The sensor accurately states whether it will rain or not, i.e.,
     P(Sensor predicts "Sunshine" | Weather will actually be Sunny) = 1
   - In the stock market example,
     P(Expert says "Market Up" | Market really Does Go Up) = 1
2. Also, the expert must never say that the state S will occur if any other state ($\bar{S}$) will occur.

Decision Tree without Additional Information

Decision maker's prior probabilities: Sun (0.4) and Rain (0.6).

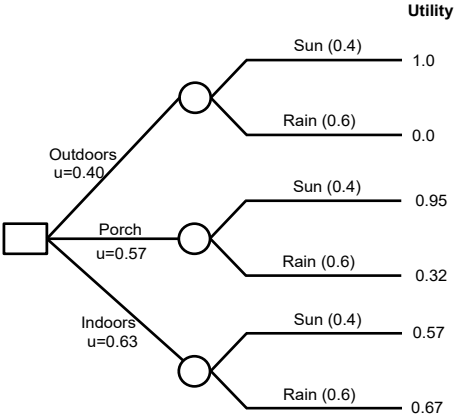


Figure: 10.7 on Page 201 (Howard and Abbas)

Decision Tree with Clairvoyance

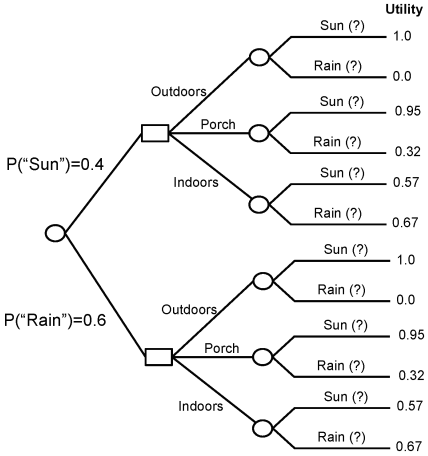


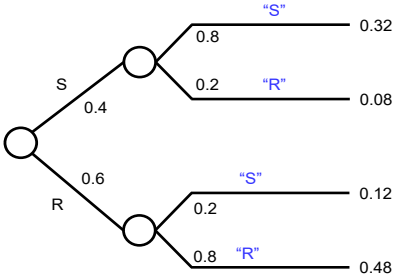
Figure: 10.8 on Page 202 (Howard and Abbas)

Expected Value of Imperfect Information

Imperfect Sensor

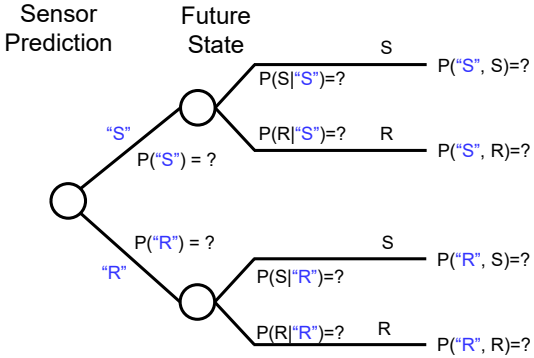
Let us say that the sensor is **not** perfect. It has 80% accuracy.

	Actual Future State	
Sensor Prediction	Sunshine (S)	Rain (R)
"S"	0.8	0.2
"R"	0.2	0.8



- P(S) = 0.4
- P(R) = 0.6
- P("S" | S) = 0.8
- P("R" | R) = 0.8
- P("S" | R) = 0.2
- P("R" | S) = 0.2

Uncertainties When Using the Sensor



Flipping the Probabilities Using Bayes' Theorem

Bayes Theorem

$$P(AB) = P(A|B)P(B) = P(B|A)P(A)$$

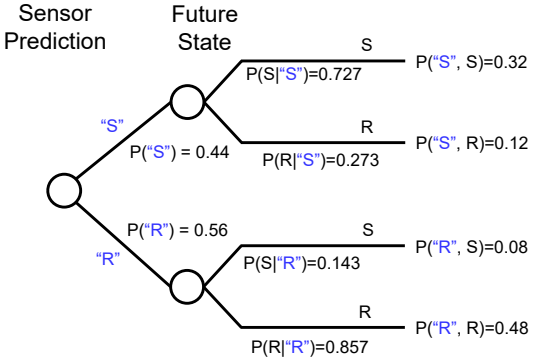
$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

Applying this to the sensor example,

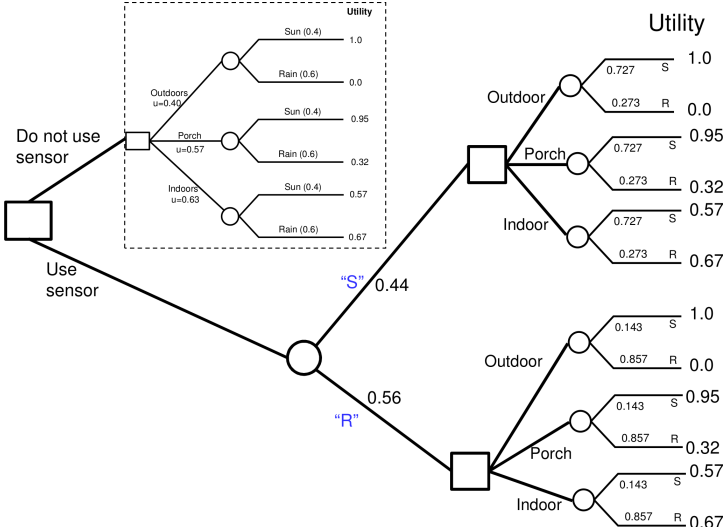
$$\begin{aligned} P(S|\text{“}S\text{”}) &= \frac{P(\text{“}S\text{”}|S)P(S)}{P(\text{”}S\text{”})} \\ &= \frac{P(\text{“}S\text{”}|S)P(S)}{P(\text{“}S\text{”}|S)P(S) + P(\text{”}S\text{”}|R)P(R)} \\ &= \frac{(0.8)(0.4)}{(0.8)(0.4) + (0.2)(0.6)} \\ &= 0.727 \end{aligned}$$

Similarly, P(R | “R”) = 0.857, P(R | “S”) = 0.273, P(S | “R”) = 0.143, P(“S”) = 0.44, P(“R”) = 0.56.

Flipping the Probabilities Using Bayes' Theorem



Including Decisions in the Tree



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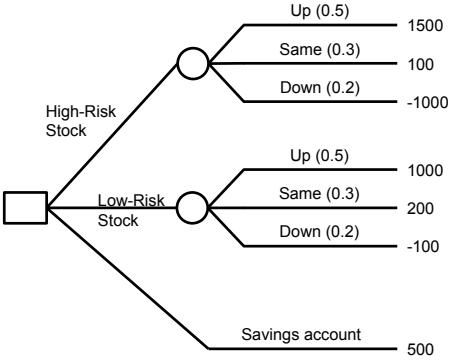

Figure: 12.1 on Page 436 (Clemen)

Example of Stock Market Investment

Deciding whether to hire the clairvoyant consultant

The decision maker has not yet consulted the clairvoyant. He is considering whether or not to consult!

There is 50% chance that the clairvoyant would say that the market will go up, 30% chance that the market will stay flat, and 20% chance that the market will go down.

Expected Value of Perfect Information

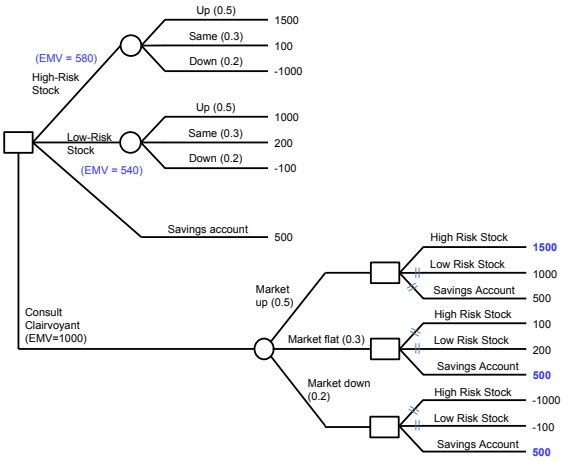

Figure: 12.3 on Page 440 (Clemen)

Expected Value of "Imperfect" Information

Suppose that the expert's track record shows that if the market actually will rise, he says:

- "Up" 80% of the time
- "Flat" 10% of the time
- "Down" 10% of the time

	True Market State		
Economist's Prediction	Up	Flat	Down
"Up"	0.80	0.15	0.20
"Flat"	0.10	0.70	0.20
"Down"	0.10	0.15	0.60
	1.00	1.00	1.00

Decision Tree for the Investment Example

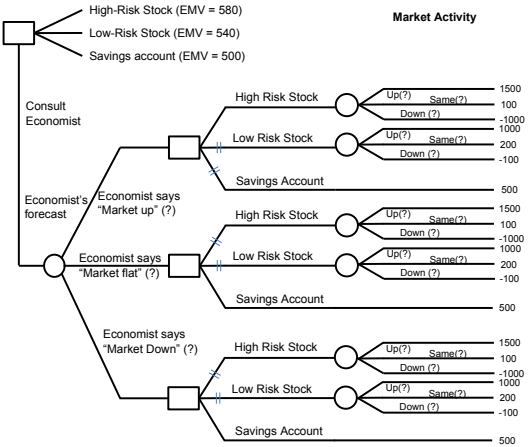

Figure: 12.5 on Page 443 (Clemen)

Flipping the Probability Tree

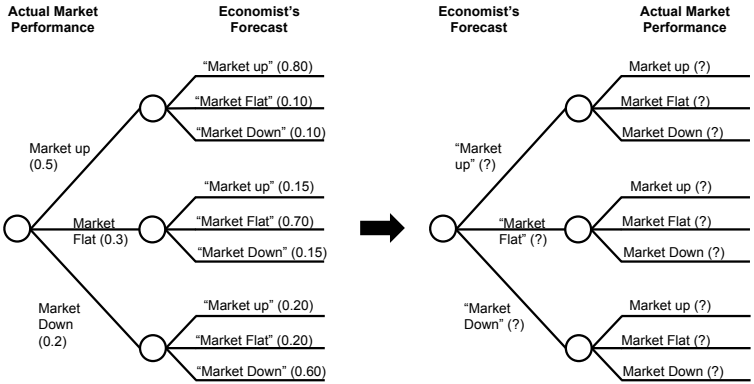

Figure: 12.7 on Page 444 (Clemen)

Using Bayes' Theorem to Flip Probabilities

$$
\begin{aligned}
&P(\text{Market Up}|\text{Economist Says "Up"}) \\
&= P(\text{Up}|\text{"Up"}) \\
&= \frac{P(\text{"Up"}|\text{Up})P(\text{Up})}{P(\text{"Up"}|\text{Up})P(\text{Up}) + P(\text{"Up"}|\text{Flat})P(\text{Flat}) + P(\text{"Up"}|\text{Down})P(\text{Down})} \\
&= \frac{0.8(0.5)}{0.8(0.5) + 0.15(0.3) + 0.2(0.2)} \\
&= \frac{0.400}{0.485} \\
&= 0.8247
\end{aligned}
$$

Posterior Probabilities For Market Trends

	Posterior Probability for			
Economist's Prediction	Market Up	Market Flat	Market Down	
"Up"	0.8247	0.0928	0.0825	1.0000
"Flat"	0.1667	0.7000	0.1333	1.0000
"Down"	0.2325	0.2093	0.5581	1.0000

Completed Decision Tree

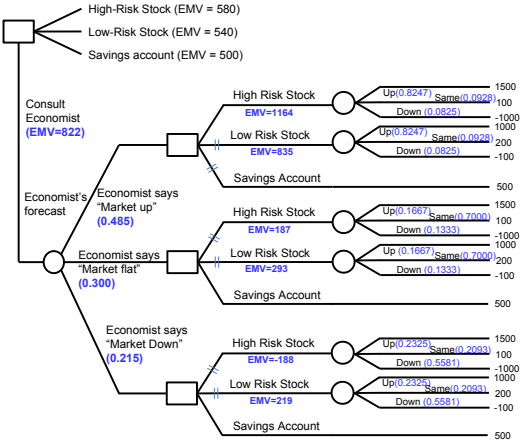

Figure: 12.8 on Page 446 (Clemen)

Summary

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