Module 09
Value of Information

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Decision Making in Engineering Design

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Module Outline

1. Introduction to Value of Information
   - Information Acquisition Decisions in Design
   - Illustrative Examples
   - Key Concepts

2. Expected Value of Perfect Information

3. Expected Value of Imperfect Information
   - 1. Rain Sensor Example
   - 2. Stock Market Example

Introduction to Value of Information
Sources of Information in Engineering Design

Information Sources:
- Consultants
- Simulation Models
- Experiments
- ...

Information is costly!
Should you acquire information (or source of information)? If so, how much should you pay for it?
Is Further Refinement Necessary? A Conceptual Example

How much refinement of a simulation model is appropriate for design?

Should this model be refined further?

Simulation model predicts material strength

Material 1

Decision

Material 2

Objective: Maximize the pressure that the vessel can withstand

Selected Material Alternative

Inferences

• Improved accuracy does not imply improved usefulness in design!!!

• The appropriateness depends on the overall design problem (constraints, objectives, and variable bounds)
All models are wrong, some models are useful! [George Box]
Questions to be addressed:

- What is an appropriate basis on which to evaluate the **value of information** in a decision situation?
- What does it mean for an expert to provide **perfect information**?
- How does probability relate to the idea of information?
Illustrative Example (1)
Buying a Rain Predictor

Three alternatives:
- Party outdoors
- Party on porch
- Party indoors

Uncertainty: Payoff depends on whether it is sunny or it rains.

Information acquisition decision: Whether to purchase a rain predicting sensor.

Figure: 10.7 on Page 201 (Howard and Abbas)
Illustrative Example (2)
Investing in the Stock Market

Investor has three alternatives:
- High-risk stock
- Low-risk stock
- Savings account

**Uncertainty**: Payoff on stocks depends on whether the market goes up, remains same, or goes down.

**Information acquisition decision**: Whether to get expert advice.

*Figure: 12.1 on Page 436 (Clemen)*
If the decision maker will make the same decision regardless of what the new information is, then the information has no value!

Rain Prediction Sensor Example:
- The best decision without the sensor is to hold the party indoors.
- If after buying the sensor, the decision still remains the same (irrespective of what the sensor shows) ⇒ no value.
Value of information needs to be determined before actually getting the information (e.g., before hiring the expert).

- **Worst case scenario**: Decision remains the same even after hiring the expert ⇒ Zero value of information.
- **Better scenarios**: Expected value increases ⇒ Positive value of information.
- **Best case scenario**: Perfect Information (resolving all uncertainty; Expert tells us exactly what will happen) ⇒ Maximum value of information (i.e., Expected Value of Perfect Information).
Expected Value of Perfect Information
Clairvoyant: An expert’s information is said to be perfect if it is always correct.

1. When state $S$ will occur, the information source always says so.
   - The sensor accurately states whether it will rain or not, i.e.,
     \[ P(\text{Sensor predicts "Sunshine" | Weather will actually be Sunny}) = 1 \]
   - In the stock market example,
     \[ P(\text{Expert says "Market Up" | Market really Does Go Up}) = 1 \]

2. Also, the expert must never say that the state $S$ will occur if any other state ($\bar{S}$) will occur.
Decision Tree without Additional Information

Decision maker’s prior probabilities: Sun (0.4) and Rain (0.6).

Utility

Outdoors: u = 0.40
- Sun (0.4) → 1.0
- Rain (0.6) → 0.0

Porch: u = 0.57
- Sun (0.4) → 0.95
- Rain (0.6) → 0.32

Indoors: u = 0.63
- Sun (0.4) → 0.57
- Rain (0.6) → 0.67

Figure: 10.7 on Page 201 (Howard and Abbas)
Introduction to Value of Information

Expected Value of Perfect Information

Expected Value of Imperfect Information

Figure: 10.8 on Page 202 (Howard and Abbas)
Expected Value of Imperfect Information
Let us say that the sensor is not perfect. It has 80% accuracy.

<table>
<thead>
<tr>
<th>Sensor Prediction</th>
<th>Sunshine (S)</th>
<th>Rain (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>“S”</td>
<td>0.8</td>
<td>0.2</td>
</tr>
<tr>
<td>“R”</td>
<td>0.2</td>
<td>0.8</td>
</tr>
</tbody>
</table>

- $P(S) = 0.4$
- $P(R) = 0.6$
- $P(\text{“S”} \mid S) = 0.8$
- $P(\text{“R”} \mid R) = 0.8$
- $P(\text{“S”} \mid R) = 0.2$
- $P(\text{“R”} \mid S) = 0.2$
Uncertainties When Using the Sensor

Sensor Prediction | Future State
--- | ---
S | S
R | R

P("S") = ?
P("R") = ?
P(S|"S") = ?
P(R|"S") = ?
P(S|"R") = ?
P(R|"R") = ?
P("S", S) = ?
P("S", R) = ?
P("R", S) = ?
P("R", R) = ?
Flipping the Probabilities Using Bayes’ Theorem

Bayes Theorem

\[ P(AB) = P(A|B)P(B) = P(B|A)P(A) \]

\[ P(B|A) = \frac{P(A|B)P(B)}{P(A)} \]

Applying this to the sensor example,

\[ P(S|"S") = \frac{P(\"S"|S)P(S)}{P(\"S")} \]

\[ = \frac{P(\"S"|S)P(S)}{P(\"S"|S)P(S) + P(\"S"|R)P(R)} \]

\[ = \frac{(0.8)(0.4)}{(0.8)(0.4) + (0.2)(0.6)} \]

\[ = 0.727 \]

Similarly, \( P(R|"R") = 0.857, P(R|"S") = 0.273, P(S|"R") = 0.143, \)

\( P("S") = 0.44, P("R") = 0.56. \)
Flipping the Probabilities Using Bayes’ Theorem

Sensor Prediction | Future State
---|---
S | P(S|“S”)=0.727
P(“S”, S)=0.32
P(“S”, R)=0.12
R | P(R|“S”)=0.273

“S” | P(“S”) = 0.44
“R” | P(“R”) = 0.56

P(S|“R”)=0.143
P(R|“R”)=0.857
P(“R”, S)=0.08
P(“R”, R)=0.48

S | P(R|“S”)=0.273
P(R|“R”)=0.857

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Including Decisions in the Tree

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Investor has three alternatives:

- High-risk stock
- Low-risk stock
- Savings account

**Uncertainty**: Payoff on stocks depends on whether the market goes up, remains same, or goes down.

**Information acquisition decision**: Whether to get expert advice.

*Figure: 12.1 on Page 436 (Clemen)*
The decision maker has not yet consulted the clairvoyant. He is considering whether or not to consult!

There is 50% chance that the clairvoyant would say that the market will go up, 30% chance that the market will stay flat, and 20% chance that the market will go down.
Expected Value of Perfect Information

High-Risk Stock
(EMV = 580)

Low-Risk Stock
(EMV = 540)

Savings account
500

Consult Clairvoyant (EMV=1000)

Market up (0.5)

Up (0.5)
1500
Same (0.3)
100
Down (0.2)
-1000

Up (0.5)
1000
Same (0.3)
200
Down (0.2)
-100

Savings account
500

Figure: 12.3 on Page 440 (Clemen)
Expected Value of “Imperfect” Information

Suppose that the expert’s track record shows that if the market actually will rise, he says:

- “Up” 80% of the time
- “Flat” 10% of the time
- “Down” 10% of the time

<table>
<thead>
<tr>
<th>True Market State</th>
<th>Up</th>
<th>Flat</th>
<th>Down</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economist’s Prediction</td>
<td>0.80</td>
<td>0.15</td>
<td>0.20</td>
</tr>
<tr>
<td>“Up”</td>
<td>0.10</td>
<td>0.70</td>
<td>0.20</td>
</tr>
<tr>
<td>“Flat”</td>
<td>0.10</td>
<td>0.15</td>
<td>0.60</td>
</tr>
<tr>
<td>“Down”</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Decision Tree for the Investment Example

- **Consult Economist**
  - **Economist’s forecast**
    - **Economist says “Market up” (?)**
      - High-Risk Stock (EMV = 580)
      - Low-Risk Stock (EMV = 540)
      - Savings account (EMV = 500)
    - **Economist says “Market flat” (?)**
      - Savings Account
    - **Economist says “Market Down” (?)**
      - High-Risk Stock
      - Low-Risk Stock
      - Savings Account

**Market Activity**

- **Up(?) Same(?)**
  - High Risk Stock
  - Low Risk Stock
  - Savings Account
  - Values: 1500, 100, 200, -100
- **Down (?)**
  - High Risk Stock
  - Low Risk Stock
  - Savings Account
  - Values: 1000, 200, -100

**Figure: 12.5 on Page 443 (Clemen)**
Flipping the Probability Tree

Actual Market Performance

- Market up (0.5)
  - Market Flat (0.3)
    - Market Down (0.2)

Economist's Forecast

- "Market up" (0.80)
  - "Market Flat" (0.10)
  - "Market Down" (0.10)
- "Market up" (0.15)
  - "Market Flat" (0.70)
  - "Market Down" (0.15)
- "Market up" (0.20)
  - "Market Flat" (0.20)
  - "Market Down" (0.60)

Economist's Forecast

- "Market up" (?)
  - Market Flat (?)
  - Market Down (?)
- "Market Flat" (?)
  - Market Flat (?)
  - Market Down (?)
- "Market Down" (?)
  - Market Flat (?)
  - Market Down (?)

Actual Market Performance

- Market up (?)
- Market Flat (?)
- Market Down (?)

Figure: 12.7 on Page 444 (Clemen)
Using Bayes’ Theorem to Flip Probabilities

\[ P(\text{Market Up}|\text{Economist Says “Up”}) \]

\[ = P(\text{Up}|“Up”) \]

\[ = \frac{P(“Up”|\text{Up})P(\text{Up})}{P(“Up”|\text{Up})P(\text{Up}) + P(“Up”|\text{Flat})P(\text{Flat}) + P(“Up”|\text{Down})P(\text{Down})} \]

\[ = \frac{0.8(0.5)}{0.8(0.5) + 0.15(0.3) + 0.2(0.2)} \]

\[ = \frac{0.400}{0.485} \]

\[ = 0.8247 \]
### Posterior Probabilities For Market Trends

<table>
<thead>
<tr>
<th>Economist’s Prediction</th>
<th>Market Up</th>
<th>Market Flat</th>
<th>Market Down</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Up&quot;</td>
<td>0.8247</td>
<td>0.0928</td>
<td>0.0825</td>
</tr>
<tr>
<td>&quot;Flat&quot;</td>
<td>0.1667</td>
<td>0.7000</td>
<td>0.1333</td>
</tr>
<tr>
<td>&quot;Down&quot;</td>
<td>0.2325</td>
<td>0.2093</td>
<td>0.5581</td>
</tr>
</tbody>
</table>
1. Rain Sensor Example
2. Stock Market Example

**Completed Decision Tree**

- High-Risk Stock (EMV = 580)
- Low-Risk Stock (EMV = 540)
- Savings account (EMV = 500)

**Consult Economist (EMV=822)**

- Economist’s forecast
  - Economist says "Market up" (0.485)
  - Economist says "Market flat" (0.300)
  - Economist says "Market Down" (0.215)

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**09: Value of Information**

**Figure: 12.8 on Page 446 (Clemen)**
Summary

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   - Information Acquisition Decisions in Design
   - Illustrative Examples
   - Key Concepts

2. Expected Value of Perfect Information

3. Expected Value of Imperfect Information
   - 1. Rain Sensor Example
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References
