Module 08
Multiattribute Utility Theory:
Example Application in Design and Manufacturing

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Module Outline

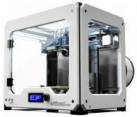
1. 3D Printing Application

2. Example: Light Switch Cover Plate
   - 1. Assessing the Multi-attribute Utility Function
   - 2. Choosing Alternative based on the Utility Function

Fernandez, M.G., Seepersad, C.C., Rosen, D.W., Allen, J.K., and Mistree, F., 2005, "Decision Support in Concurrent Engineering – The Utility-Based Selection Decision Support Problem,", *Concurrent Engineering*, Vol. 13, No. 1, 13-27.

3D Printing Application

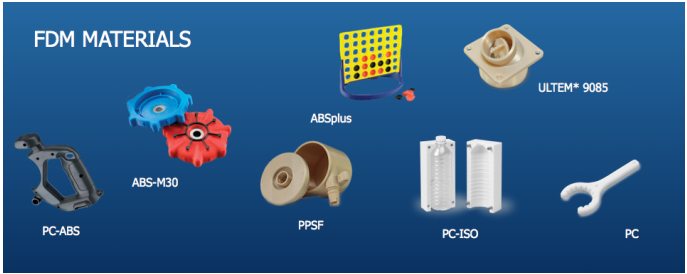
3D Printing Processes




- FDM: Fused Deposition Modeling
- Polyjet
- SLA: Stereolithography
- SLS: Selective Laser Sintering
- DMLS: Direct Metal Laser Sintering

Fused Deposition Modeling (FDM)

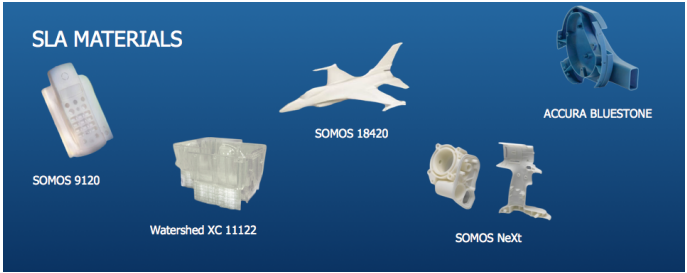
Fused Deposition Modeling is an additive manufacturing process which is used to build concept models, functional prototypes and end-use parts with engineering-grade and high accuracy thermoplastics. The FDM technology is designed to create with production-grade thermoplastic materials that possess the most mechanical, thermal and chemical strength.



Source: http://proto3000.com/fdm-thermoplastic-materials.php

Stereolithography (SLA)

Stereolithography is one of the most advanced and accurate rapid prototyping technologies, which is most ideal for concept modeling, fit-form-function testing and a variety of molding techniques. In an SLA process, the laser is capable of fusing metal, plastic, glass and ceramic particles into a 3D Printed model. Offering a variety of different coloured materials, SLA is an ideal technology for economical and rapid 3D printing.



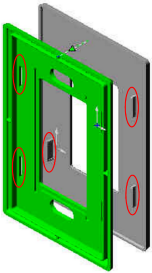
Source: http://proto3000.com/sla-photopolymer-materials.php

Diverse Applications of 3D Printing

- Concept Modeling
- Display Parts Finishing
- Fastening and Gluing
- Fit and Form Testing
- Functional Testing
- Heat Resistance Testing
- Manufacturing-Tooling
- Medical and Dental Tooling
- Rubber Part Testing
- Shape Changing
- Surface Improvement

Example: Light Switch Cover Plate

Example: Light Switch Cover Plate Assembly



Objectives of the Prototype:

- functional product validation, particularly with respect to the snap-fitting of components,
- determining closeness of fit or tolerance of the two interfacing components,
- obtaining a feel for the product, and
- visual and physical confirmation of 3D interface integrity

Identifying Attributes

1. Tensile strength: The strength of the material under tension
2. Young's modulus: The modulus of elasticity of the material
3. Flexural strength: The strength of the material under bending
4. Flexural modulus: The modulus of flexural stiffness of the material
5. Detail capability: The ability to reproduce the part in sufficient detail.
6. Accuracy: The ability to reproduce the part with sufficient accuracy (maintaining dimensional requirements)
7. Elongation at break
8. Hardness
9. Density
10. Heat Deflection
11. Durability
12. Build Time
13. Cost

Assessment Procedure for Multiattribute Utility Functions

1. Introducing the terminology and ideas.
2. Identifying relevant independence assumptions.
3. Assessing conditional utility functions or isopreference curves.
4. Assessing the scaling constants.
5. Checking for consistency and reiterating.

Refer to Modules 6 and 7.

Step 2: Identifying Relevant Independence Assumptions

Goal: To simplify utility assessment.

Determine whether one (or more) of these conditions is (are) satisfied.

1. Preferential Independence
2. Conditional Utility Independence
3. Mutual Utility Independence
4. Additive Independence

Rest of the slides are based on the assumption that Additive Independence condition is satisfied, i.e.,

$$U(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} k_i u_i(x_i)$$

Refer to Module 7.

Step 3: Assessing Conditional Utility Functions, $u_i(x_j)$

Procedure:

1. Preparing for assessment
2. Identifying the relevant qualitative characteristics
3. Specifying quantitative restrictions
4. Choosing a utility function
5. Checking for consistency

Step 3: Assessing Conditional Utility Functions

Qualitative Characteristics

Attribute: Tensile Strength

Monotonicity: Non-monotonic. Utility is maximum at the target value.

- Lower Unacceptable: 50 MPa
- Ideal: 65 MPa
- Upper Unacceptable: 75 MPa

Risk attitude: Risk averse

Step 3: Assessing Conditional Utility Functions

Quantitative Characteristics

The chosen functional form: $u(x) = a + bx + ce^{dx}$

For this function, the risk measure,

$$r(x) = -\frac{u''(x)}{u'(x)} = -\frac{d^2 e^{dx}}{\left(\frac{b}{c} + de^{dx}\right)}$$

Question

Under which conditions does this function model risk aversion?

Step 3: Assessing Conditional Utility Functions

Quantitative Characteristics

Tensile Strength:

- Lower Unacceptable: 50 MPa
- Ideal: 65 MPa
- Upper Unacceptable: 75 MPa

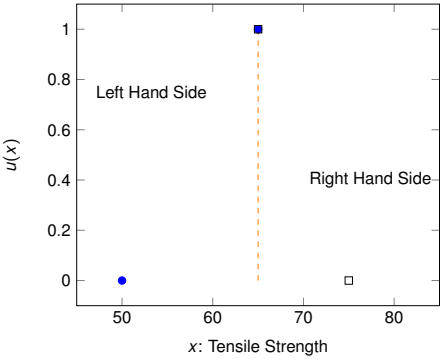
Split the utility function into two parts and set scale:

- Left Side Utility Function:

$$u_L(50) = 0, u_L(65) = 1$$

- Right Side Utility Function:

$$u_R(65) = 1, u_R(75) = 0$$



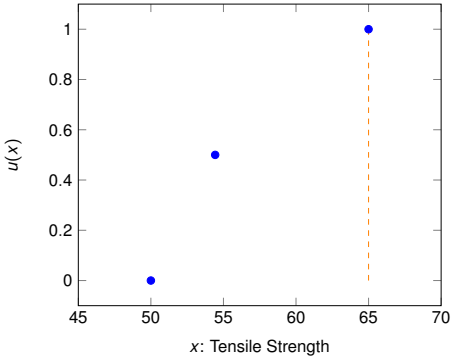
Step 3: Assessing Conditional Utility Functions

Quantitative Characteristics

Lottery question 1:

$$\langle 50, 0.5, 65 \rangle \sim \boxed{x_{0.5}}$$

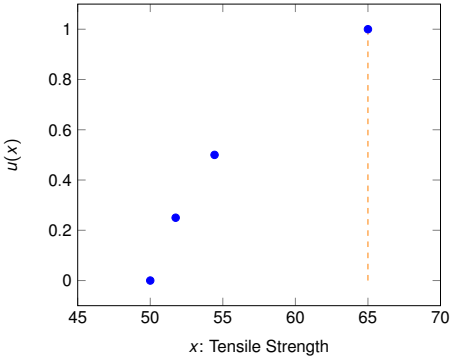
Say answer is: $x_{0.5} = 54.43$



Lottery question 2:

$$\langle 50, 0.5, 54.43 \rangle \sim \boxed{x_{0.25}}$$

Say answer is: $x_{0.25} = 51.75$



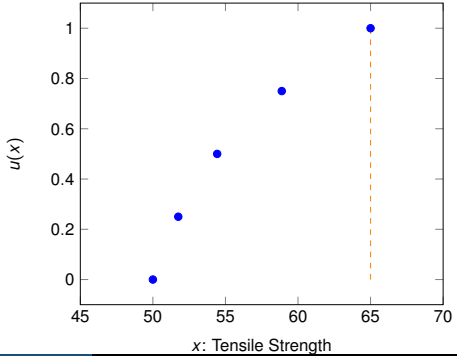
Step 3: Assessing Conditional Utility Functions

Quantitative Characteristics

Lottery question 3:

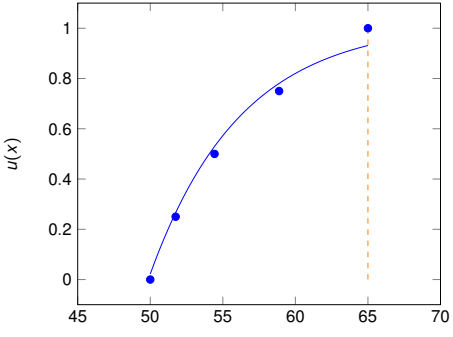
$$\langle 54.43, 0.5, 65 \rangle \sim \boxed{x_{0.75}}$$

Say answer is: $x_{0.75} = 58.88$



Curve Fit: $u(x) = a + bx + ce^{dx}$

$$u_L(x) = 1.022 - 2981e^{-0.16x}$$

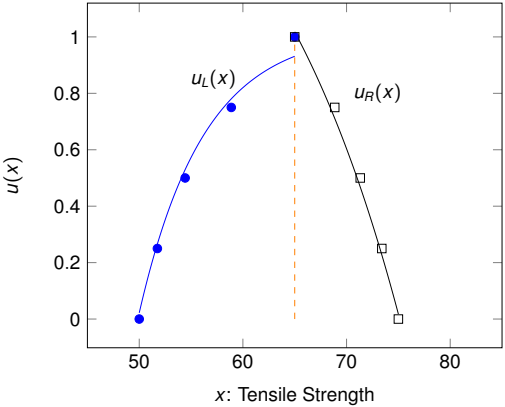


Step 3: Assessing Conditional Utility Functions

Quantitative Characteristics

$$u_L(x) = 1.022 - 2981e^{-0.16x}$$

$$u_R(x) = 2.018 - 0.01103e^{0.06934x}$$



Step 4: Assessing the Scaling Constants

Additive Multi-attribute Utility Function,

$$U(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} k_i u_i(x_i)$$

Approach: To find k_i, use indifference questions such as:

$$(TS_0, YM_{0.55}, FS_{0.55}, FM_{0.55}, DC_{0.55}, A_{0.55})$$

$$\sim$$

$$(\boxed{TS_?}, YM_{0.45}, FS_{0.45}, FM_{0.45}, DC_{0.45}, A_{0.45})$$

Need $(N-1)$ such questions to estimate k_i.

Note: This is not a lottery question.

Step 4: Assessing the Scaling Constants

Outcome

Attribute	***k*-value**
Tensile Strength	$k_1 = 0.193741$
Young's Modulus	$k_2 = 0.1864461$
Flexural Strength	$k_3 = 0.189041$
Flexural Modulus	$k_4 = 0.1922651$
Detail Capability	$k_5 = 0.1563661$
Accuracy	$k_6 = 0.0821421$

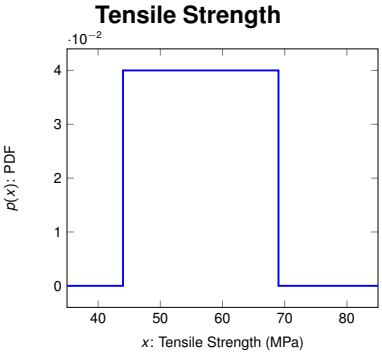
Identify Alternatives

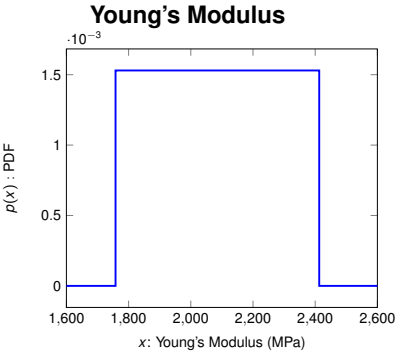
Process-Material Combinations:

1. SLA250 with DSM7110 (by DSM Somos®)
2. SLA3500 with SL7510 (by Vantico AG)
3. SLA3500 with DSM8120 (by DSM Somos®)
4. FDM1650 with P400 (by Stratasys Inc.)
5. MJM2100 with TJ75 (by 3D Systems)

Probability Distributions of Attributes for an Alternative

Example: SLA250-DSM7110





Determine these for each attribute for each alternative!

Evaluate the Expected Utility for Each Alternative

For the additive multi-attribute utility function,

$$U(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} k_i u_i(x_i)$$

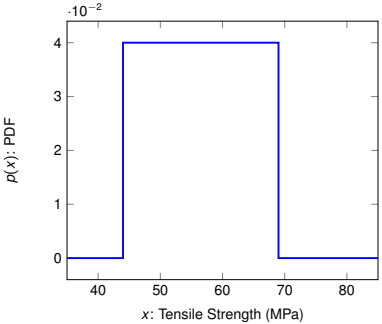
The expected utility,

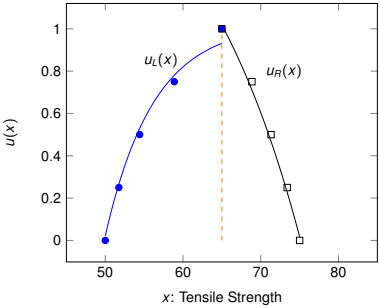
$$E[U(x_1, x_2, \ldots, x_N)] = \sum_{i=1}^{N} k_i E[u_i(x_i)]$$

Therefore, first calculate the expectation of the single-attribute utility functions, and then scale them.

Evaluate the Expected Utility for Each Alternative

Tensile Strength for SLA250-DSM7110





$$E[u(x)] = \int p(x)u(x)dx$$

Denote this as: $E[u_1(TS)]$

Evaluate the Expected Utility for Each Alternative

Expected Utility for an Alternative:

$$
\begin{aligned}
E[U(x_1, x_2, \ldots, x_N)] = \sum_{i=1}^{N} k_i E[u_i(x_i)] \\
= & k_1 E[u_1(TS)] + k_2 E[u_2(YM)] \\
& + k_3 E[u_3(FS)] + k_4 E[u_4(FM)] \\
& + k_5 E[u_5(DC)] + k_6 E[u_6(A)]
\end{aligned}
$$

Results: Expected Utility

Alternative	Expected Utility
SLA250-DSM7110	0.62214
SLA3500-SL7510	0.44195
SLA3500-DSM8120	0
FDM1650-P400	0
MJM2100-TJ75	0

Choose the alternative with the maximum expected utility!

Summary

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2. Example: Light Switch Cover Plate
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