AC-DC converters (Rectifiers)

The objective of an (AC-DC converter) / rectifier is to produce a voltage that is purely dc or has some specified dc components.

In many applications, a rectifier serves as the input voltage to be followed by a dc-dc converter or a dc-ac converter.

Ideally, the output voltage of a rectifier is expected to be purely dc. However, in practice, harmonics appear as voltage ripples.

We will study the diode-rectifiers which are the most popular rectifiers:

- Single-phase H-Bridge (Full Bridge) Rectifiers
- Three-phase Rectifiers
Single-phase Diode Bridge Rectifiers with pure resistive load

For the bridge rectifier, there are the following basic observations:

1) If \( V_{\text{in}} > 0 \) → \( D_1 \& D_4 : \text{on} \)
   \( D_2 \& D_3 : \text{off} \)
   \[ V_0 = V_{\text{in}} \]
   \[ I_0 = \frac{V_{\text{in}}}{R} \]

2) If \( V_{\text{in}} < 0 \) → \( D_1 \& D_4 : \text{off} \)
   \( D_2 \& D_3 : \text{on} \)
   \[ V_0 = -V_{\text{in}} \]
   \[ I_0 = -\frac{V_{\text{in}}}{R} \]
For a sinusoidal input voltage \( V_{\text{in}} = V_m \sin \theta \)

- \( V_o = V_{\text{in}} \)
- \( V_o = -V_{\text{in}} \)

\[ i_o = \frac{V_{\text{in}}}{R} \]
\[ i_o = -\frac{V_{\text{in}}}{R} \]

\[ \frac{V_m}{R} \]

\[ \frac{i_o}{R} \]

\[ < V_{\text{out}} > = \frac{1}{\pi} \int_0^\pi V_m \sin \theta \, d\theta = \frac{2V_m}{\pi} \]

The desired dc voltage component

\[ < i_{\text{out}} > = \frac{< V_{\text{out}} >}{R} = \frac{2V_m}{RR} \]

However, as it can be seen from \( V_o \), it contains a dc component (calculated as \( \frac{2V_m}{\pi} \)) and some harmonics.

Let's look at the Fourier series to analyze the harmonics.
\[ V_o = V_{out} + \sum_{i=1}^{\infty} a_n \cos(n\theta) + \sum_{i=1}^{\infty} b_n \sin(n\theta) \]

\[ b_n = \frac{2}{\pi} \int_{0}^{\pi} V_o \sin(n\theta) \, d\theta = \frac{2}{\pi} \int_{0}^{\pi} V_m \sin\theta \cdot \sin(n\theta) \, d\theta \]

\[ = \frac{2V_m}{\pi} \cdot \frac{1}{2} \left[ \int_{0}^{\pi} (\cos((n+1)\theta) - \cos((n-1)\theta)) \, d\theta \right] \]

\[ = \frac{V_m}{\pi} \left[ \sin((n+1)\pi) - \sin((n-1)\pi) \right] = 0 \]

\[ a_n = \frac{2}{\pi} \int_{0}^{\pi} V_o \cos(n\theta) \, d\theta = \frac{2}{\pi} \int_{0}^{\pi} V_m \sin\theta \cdot \cos(n\theta) \, d\theta \]

\[ = \frac{2V_m}{\pi} \cdot \frac{1}{2} \left[ \int_{0}^{\pi} \sin((n+1)\theta) - \sin((n-1)\theta) \, d\theta \right] \]

\[ = \frac{V_m}{\pi} \left[ \left. -\frac{\cos((n+1)\theta)}{n+1} \right|_{0}^{\pi} + \frac{\cos((n-1)\theta)}{n-1} \right]^{\pi}_{0} \]

\[ = \frac{V_m}{\pi} \left( \frac{1 - \cos((n+1)\pi)}{n+1} + \frac{1 - \cos((n-1)\pi)}{n-1} \right) \]
if $n$: even, $a_n = \frac{2V_m}{\pi} \left( \frac{1}{n+1} - \frac{1}{n-1} \right)$

if $n$: odd, $a_n = 0$

$$V_{out} = \frac{2V_m}{\pi} + \sum_{n=2,4,\ldots}^{\infty} \frac{2V_m}{\pi} \left( \frac{1}{n+1} - \frac{1}{n-1} \right) \cos(n\theta)$$

$$= \sum_{n=2,4,\ldots}^{\infty} \frac{2V_m}{\pi} \left( \frac{1}{n-1} - \frac{1}{n+1} \right) \cos(n\theta + \pi)$$

$$i_{out} = \frac{V_{out}}{R}, \quad I_{out} = \langle i_{out} \rangle = \langle \frac{V_{out}}{R} \rangle = \langle \frac{V_0}{R} \rangle = \frac{2V_m}{R}$$

$$i_n = \frac{V_n}{R} = \sum_{n=2,4,\ldots}^{\infty} \frac{2V_m}{RR} \left( \frac{1}{n-1} - \frac{1}{n+1} \right) \cos(n\theta + \pi)$$

Since the average voltage (dc component) at the output of a diode-bridge rectifier is equal to $\langle V_{out} \rangle = \frac{2V_m}{\pi}$, it is called an uncontrolled bridge rectifier. The voltage is a function of AC-source voltage amplitude ($V_m$) which is usually fixed. The output voltage of Rectifier is fixed and not controllable!
If the load is almost purely inductive, \( L \) is very large, then the output current is almost constant \( \Rightarrow i_\text{avg} = \text{dc Value} \).
Single-phase diode rectifier with series inductive-resistive load

\[ \text{if } \text{Vin} > 0 \Rightarrow D_1 \text{ & } D_4 \text{ are on } \Rightarrow V_{out} = \text{Vin} \Rightarrow i_{D_1} \text{ & } i_{D_4} = i_o \\
\text{Vin} < 0 \Rightarrow D_2 \text{ & } D_3 \text{ are on } \Rightarrow V_{out} = -\text{Vin} \Rightarrow i_{D_3} \text{ & } i_{D_2} = -i_o \\\n\Rightarrow i_{in} = -i_0 \]

Vin \hspace{1cm} V_o

\[ i_{in} \quad i_{in} \quad i_{in} \]
\[ i_{D_1} \quad i_{D_4} \quad i_{D_2} \quad i_{D_3} \]