User Guide for `deconvtv` (MATLAB Version 1.0)

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1 Introduction

`deconvtv` is a numerical algorithm for solving total variation constrained least-squares problems. The concept of the algorithm is based on an augmented Lagrangian method proposed in [1], and is a variation of the popularly known Alternating Direction Methods of Multipliers (ADMM) [2, 3]. In particular, `deconvtv` solves the following minimization problems

$$\underset{\mathbf{f}}{\text{minimize}} \quad \frac{\mu}{2}\|\mathbf{Hf}-\mathbf{g}\|_2^2 + \|\mathbf{f}\|_{TV}, \tag{1}$$

$$\underset{\mathbf{f}}{\text{minimize}} \quad \mu\|\mathbf{Hf}-\mathbf{g}\|_1 + \|\mathbf{f}\|_{TV}, \tag{2}$$

where $\mathbf{H}$ is a circulant matrix denoting a spatially invariant linear operator, μ is a regularization parameter, and and $\|\mathbf{f}\|_{TV}$ is the total variation norm of the data $\mathbf{f}$, defined as

$$\|\mathbf{f}\|_{TV} = \sum_k \sqrt{\beta_x[\mathbf{D}_x\mathbf{f}]_k^2 + \beta_y[\mathbf{D}_y\mathbf{f}]_k^2 + \beta_t[\mathbf{D}_t\mathbf{f}]_k^2}.$$

Here, $[\mathbf{f}]_k$ is the k-th entry of the vector $\mathbf{f}$. The operators $\mathbf{D}_x$, $\mathbf{D}_y$ and $\mathbf{D}_t$ are the gradient operators along the horizontal, vertical and temporal directions. The relative emphasis of $\mathbf{D}_x$, $\mathbf{D}_y$ and $\mathbf{D}_t$ can be controlled by β_x, β_y and β_t, respectively.

2 User Interface

The user interface of `deconvtv` is as follows:

```
out = deconvtv(g, h, mu, opts);
```

2.1 Input Variables

- `g`: Input image. It can be a gray-scaled image, color image, or gray-scaled video.
- `h`: Point spread function, could be a two-dimensional matrix, or a three-dimensional tensor.
- `mu`: Regularization parameter μ.
- `opts`: A structure of options (See below).

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2.2 Option Fields

All default settings are marked in $\{\cdot\}$.

- `opts.method`: Defines the method to be used, either `'l1'` or $\{$`'l2'`$\}$. If `'l2'` is chosen, then `deconvtv` solves Problem (1). If `'l1'` is chosen, then `deconvtv` solves Problem (2).
- `opts.beta`: An 1×3 vector specifying $(\beta_x, \beta_y, \beta_t)$. Default is $\{[1, 1, 0]\}$. For video deblurring and denoising, `opts.beta` can be chosen as [1, 1, 2.5].
- `opts.rho_r`: Regularization parameter to the constraint violation $\|\mathbf{u} - \mathbf{Df}\|^2$ (See [1]). Default is $\{2\}$.
- `opts.rho_o`: Regularization parameter to the constraint violation $\|\mathbf{Hf} - \mathbf{g} - \mathbf{r}\|^2$ (See [1]). Default is $\{50\}$.
- `opts.alpha`: Criteria for `rho_r` update (See [1]). Default is $\{0.7\}$.
- `opts.gamma`: Update constant for `rho_r` (See [1]). Default is $\{2\}$.
- `opts.max_itr`: Maximum number of iteration $\{20\}$.
- `opts.tol`: Tolerance level of relative change. Default is $\{$`1e-3`$\}$.
- `opts.print`: Print intermediate report, either `'true'` or `'false'`. Default is $\{$`false`$\}$.
- `opts.f`: Initial guess. Default is $\{$`g`$\}$.
- `opts.y1`, `opts.y2`, `opts.y3`: Initial guess of Lagrange multipliers for the constraint $\mathbf{u} = \mathbf{Df}$. Default is $\{\mathbf{0}\}$.
- `opts.z`: Initial guess of Lagrange multiplier for the constraint $\mathbf{r} = \mathbf{Hf} - \mathbf{g}$. Default is $\{\mathbf{0}\}$.

2.3 Output Variables

- `out.f`: Output image, or video.
- `out.itr`: total number of iterations elapsed
- `out.relchg`: final relative change
- `out.Df1`, `out.Df2`, `out.Df3`: Output image gradients
- `out.y1`, `out.y2`, `out.y3`: Output Lagrange multipliers
- `out.rho_r`: final regularization parameter

3 Examples

3.1 Image Denoising

Image Denoising Example

```
% Prepare images
f_orig  = im2double(imread('./data/wind.jpg'));
[rows cols frames] = size(f_orig);
H       = fspecial('gaussian', [9 9], 2);
g       = imfilter(f_orig, H, 'circular');
g       = imnoise(g, 'salt & pepper', 0.05);

% Setup parameters (for example)
opts.rho_r   = 5;           opts.rho_o   = 100;         opts.beta    = [1 1 0];
opts.print   = true;        opts.alpha   = 0.7;         opts.method  = 'l1';

% Setup mu
mu           = 20;

% Main routine
tic
out = deconvtv(g, H, mu, opts);
toc

% Display results
figure(1);  imshow(g);      title('input');
figure(2);  imshow(out.f);  title('recovered');
```

(a) Input Image

(b) Recovered Image

Figure 1: Example: Image denoising. Time elapsed: 2.2 seconds.

3.2 Image Deblurring

Image Deblurring Example

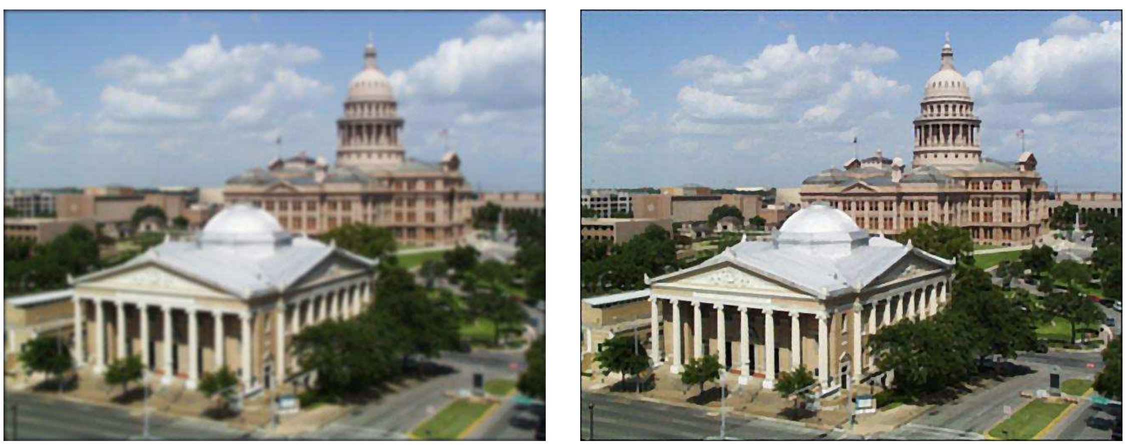
```
% Prepare images
f_orig  = im2double(imread('./data/building.jpg'));
[rows cols frames] = size(f_orig);
H       = fspecial('gaussian', [9 9], 2);
g       = imfilter(f_orig, H, 'circular');
g       = imnoise(g, 'gaussian', 0, 0.00001);

% Setup parameters (for example)
opts.rho_r   = 2;       opts.beta    = [1 1 0];     opts.print   = true;
opts.alpha   = 0.7;     opts.method  = 'l2';

% Setup mu
mu           = 10000;

% Main routine
tic
out = deconvtv(g, H, mu, opts);
toc

% Display results
figure(1);  imshow(g);      title('input');
figure(2);  imshow(out.f);  title('recovered');
```



(a) Input Image (b) Recovered Image

Figure 2: Example: Image deblurring. Time elapsed: 8.24 seconds.

3.3 Video Disparity Refinement

Video Disparity Refinement Example

```
folder_name = './data/';

fname = sprintf('%sdata%04d.jpg', folder_name, 1);
f     = im2double(imread(fname));
[rows cols frames] = size(f);
g     = zeros(rows,cols,frames);

for fidx = 1:10
    fname = sprintf('%sdata%04d.jpg', folder_name, fidx);
    f = im2double(imread(fname));
    if size(f,3)>1
        g(:,:,fidx) = rgb2gray(f);
    else
        g(:,:,fidx) = f;
    end
end

% Setup parameters (for example)
opts.beta    = [1 1 10];    opts.print   = true;    opts.method  = 'l1';

% Setup mu
mu           = 1;

% Main routine
tic
out = deconvtv(g, 1, mu, opts);
toc

% Display results
figure(1);  imshow(g(:,:,5));      title('input');
figure(2);  imshow(out.f(:,:,5));  title('recovered');
```

Figure 3: Example: Video disparity refinement. Top: Input. Bottom: Recovered. Time elapsed: 12.1 seconds for 10 frames.

4 Copyright

References

[1] S.H. Chan, R. Khoshabeh, K.B. Gibson, P.E. Gill, and T.Q. Nguyen, "An augmented Lagrangian method for total variation image restoration," *IEEE Trans. Image Process.*, vol. 20, no. 11, pp. 3097–3111, Nov. 2011.

[2] Y. Wang, J. Yang, W. Yin, and Y. Zhang, "A new alternating minimization algorithm for total variation image reconstruction," *SIAM Journal on Imaging Sciences*, vol. 1, pp. 248–272, 2008.

[3] B. Wahlberg, S. Boyd, M. Annergren, and Y. Wang, "An ADMM algorithm for a class of total variation regularized estimation problems," in *Proceedings 16th IFAC Symposium on System Identification*, Jul. 2012, vol. 16.