ECE 695 / STAT 695
Sparse Modeling and Algorithms in Statistical Learning

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What is sparse modeling?
How much can we compress an image?

$$\mathbf{x} = \sum_{i=1}^{n} c_i \varphi_i$$

Wavelet Transform
Why does wavelet work?

For most images, we need very few wavelet coefficients to “encode” the image.
What is sparsity?

A (discrete-time) signal is sparse if it has **very few** non-zeros.
What is the Mathematical Problem then?

\[ y = Ax \]

\( y \in \mathbb{R}^M \) \hspace{1cm} \text{Observed signal}

\( x \in \mathbb{R}^N \) \hspace{1cm} \text{Unknown representation coefficient (to be determined)}

\( A \in \mathbb{R}^{M \times N} \) \hspace{1cm} \text{Measurement matrix (known)}

The difficulty of the problem is when

\[ M \ll N \]

That is, we have more unknowns than equations.

If this is the case, then this \textbf{underdetermined} system has \textbf{infinitely} many solutions.
How can sparsity help?

If I know the **number** of non-zeros, and if I know **where** are these non-zeros, then

This is an over-determined system, with unique solution.

(assume columns are independent)
What is this course about?

How to solve the problem when you do **NOT** know where the zeros are.

Three parameters:

\[ M = \text{number of observations} \]
\[ N = \text{number of unknowns} \]
\[ K = \text{number of non-zeros} \]

Under WHAT conditions can we get unique solution? and How?
Applications
**Statistical Regression**

**Model Selection:** Find the support of solution

**LASSO:**
\[
\min_{x} \| Ax - y \|^2 \quad \text{subject to} \quad \| x \|_1 \leq \tau
\]

**BP-Denoise:**
\[
\min_{x} \lambda \| x \|_1 + \| Ax - y \|^2
\]

(for appropriate choice of \( \lambda \) and \( \tau \), LASSO = BPDN)
Matrix Completion

\[
\begin{equation}
\begin{aligned}
\text{minimize} \quad & ||X||_* \\
\text{subject to} \quad & A(X) = Y \\
\end{aligned}
\end{equation}
\]

\[
||X||_* = \sum_{n=1}^{N} \sigma_n(X)
\]
Communication System

\[ x \xrightarrow{\Phi} y \xrightarrow{\text{Compression Random projection}} \]

\[ \min |x| \in \mathbb{R}^N \]

https://www.ti.rwth-aachen.de/research/applications/cs.php
Single-pixel Camera

(Courtesy of Rice University)

\[ y = Ax \]

\[ A = \text{binary random matrix} \]
Image Denoising

Also for image deblurring, super-resolution, etc.

$$\minimize_{\mathbf{x}} \lambda \| \mathbf{x} \|_1 + \| A \mathbf{x} - \mathbf{y} \|_2^2$$
Course Information
Course Website:
https://engineering.purdue.edu/ChanGroup/ECE695.html
Disclaimer:

As of Jan 8, 2015,
I am still reading chapter 3 of Hastie’s book.
(There are 11 chapters of the book)
My Plan: Let’s learn together

- Reading Assignments
- Application
- Algorithm
- Theory
- Final Project
- Lecture
Course Objective

By the end of the semester, I hope you will

• Have a basic understanding of **terminologies** and **concepts** behind sparse modeling
• Be able to comment on sparse **algorithms** and do some **implementation**
• Be able to **read** some papers in the field
• Be able to **apply** some techniques to research problems
Course Work
1. Attendance (10%)

... because

• You need to learn something
• I don’t want to talk to air

• No attendance sheet because, generally, I know who’s here and who’s not here
2. Homework (25%)

• 10 papers to read
• All very popular papers
• Write me a 2-page summary
• Tell me the problem, the algorithm, and assumptions
• Tell me your comment. Pros and cons of the method
• I will test you in the mid term
3. Mid Term (25%)

• Some basic questions about the lecture
• Some basic questions about your reading assignment
4. Project (40%)

- Group size: Min: 1, Max: 2
- Proposal (individual submission)
- Report (individual submission)
- Presentation (depend on enrollment)

- Any topic involves sparse modeling and algorithm
Auditing

• You are welcome to audit (i.e., sit in) the class

• Basic requirement:
  • 2/3 attendance
  • No texting / no apps during class
  • Arrive on time / don’t leave early
  • Attend student lecture
Will I get a good grade?

• I have given F to graduate students

• No curve

• You are competing with yourself

• For a small class like this, I will be able to know everyone very well

• The more effort you spend, the better grade you will get

• Historically, so far, I have no complaint about grades.
Plagiarism

• I am serious

• If it is not your work, give proper credit

• Proposal/Report: write in your **own word even** if you work as a group (Tell me your contribution)

• If I know you plagiarized, I will take very serious actions, including:
  • Fail your course immediately
  • Report to graduate school
  • Worst case: suspend you from school

• Plagiarism is a fatal crime in academia
Emergency

• Read the emergence note

• Bad weather: If you don’t feel safe, stay home

• Civil disturbance: Stay calm, find shelter

• Sick: (Recommendation) If you are sick, stay home

• Don’t worry about “attendance” in these situations.

I understand.
Final Words

• Enjoy
• Have fun
Question?