PURDUE
ENGINEERING

ECE 695 / STAT 695
Sparse Modeling and Algorithms in Statistical Learning

Spring 2018

Stanley Chan

What is sparse modeling?

How much can we compress an image?

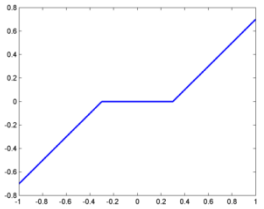
nnz = 33.51 percent

nnz = 1.21 percent

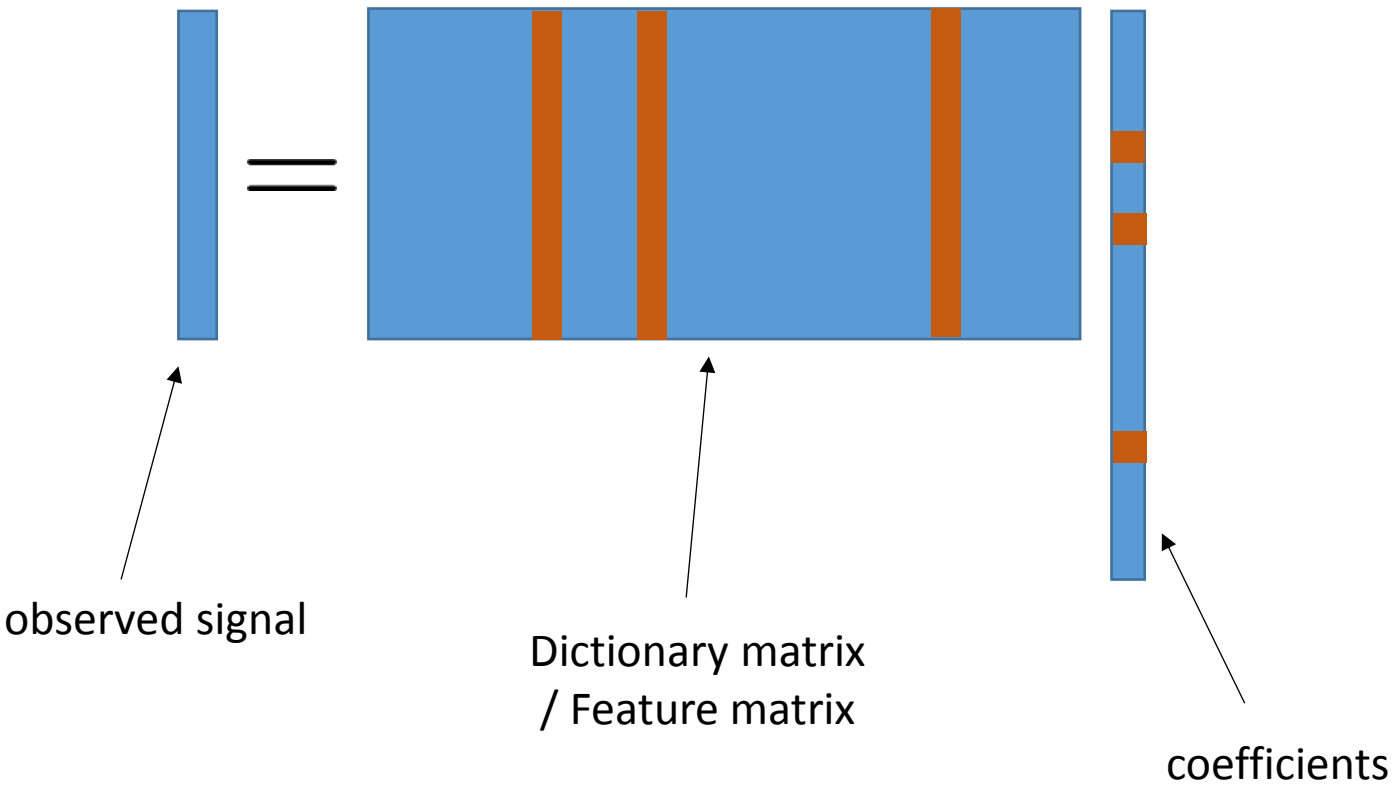
nnz = 13.58 percent

$\boldsymbol{x}$ → Wavelet Transform

$$\boldsymbol{x} = \sum_{i=1}^{n} c_i \boldsymbol{\varphi}_i$$



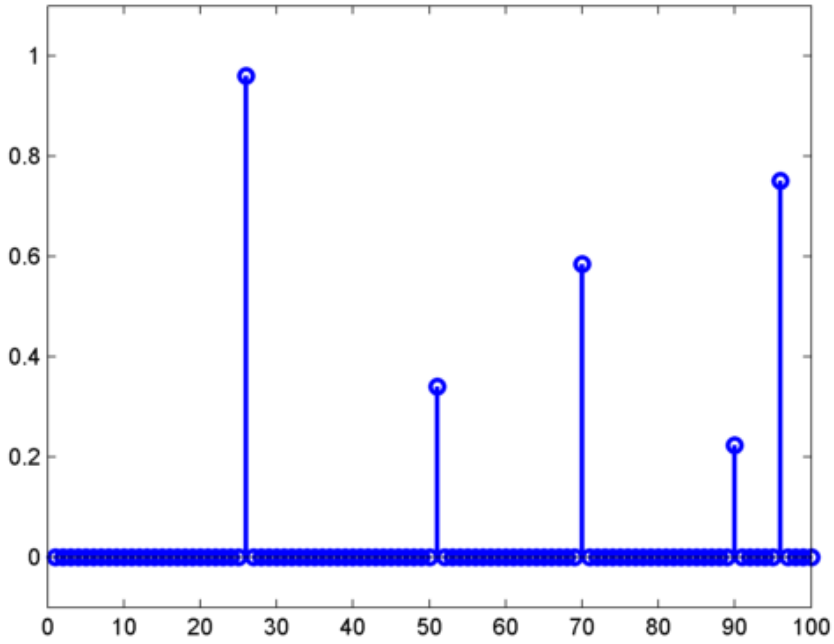
Why does wavelet work?



For most images, we need very few wavelet coefficients to “encode” the image.

What is sparsity?

A (discrete-time) signal is sparse if it has **very few** non-zeros.



What is the Mathematical Problem then?

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$$

$\boldsymbol{y} \in \mathbb{R}^M$ Observed signal

$\boldsymbol{x} \in \mathbb{R}^N$ Unknown representation coefficient (to be determined)

$\boldsymbol{A} \in \mathbb{R}^{M \times N}$ Measurement matrix (known)

The difficulty of the problem is when

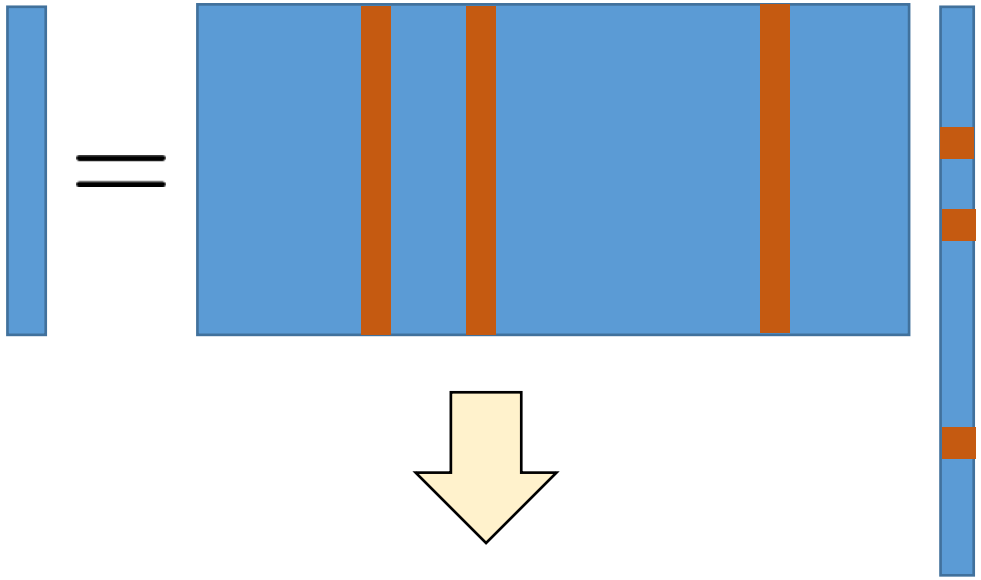
$$M \ll N$$

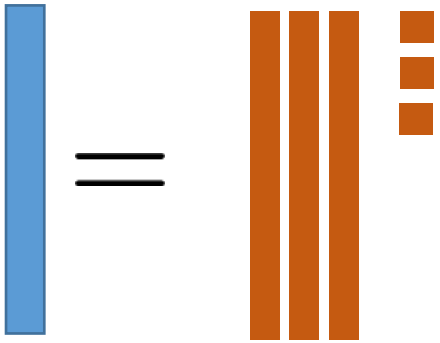
That is, we have more unknowns than equations.

If this is the case, then this **underdetermined** system has **infinitely** many solutions.

How can sparsity help?

If I know the **number** of non-zeros, and if I know **where** are these non-zeros, then





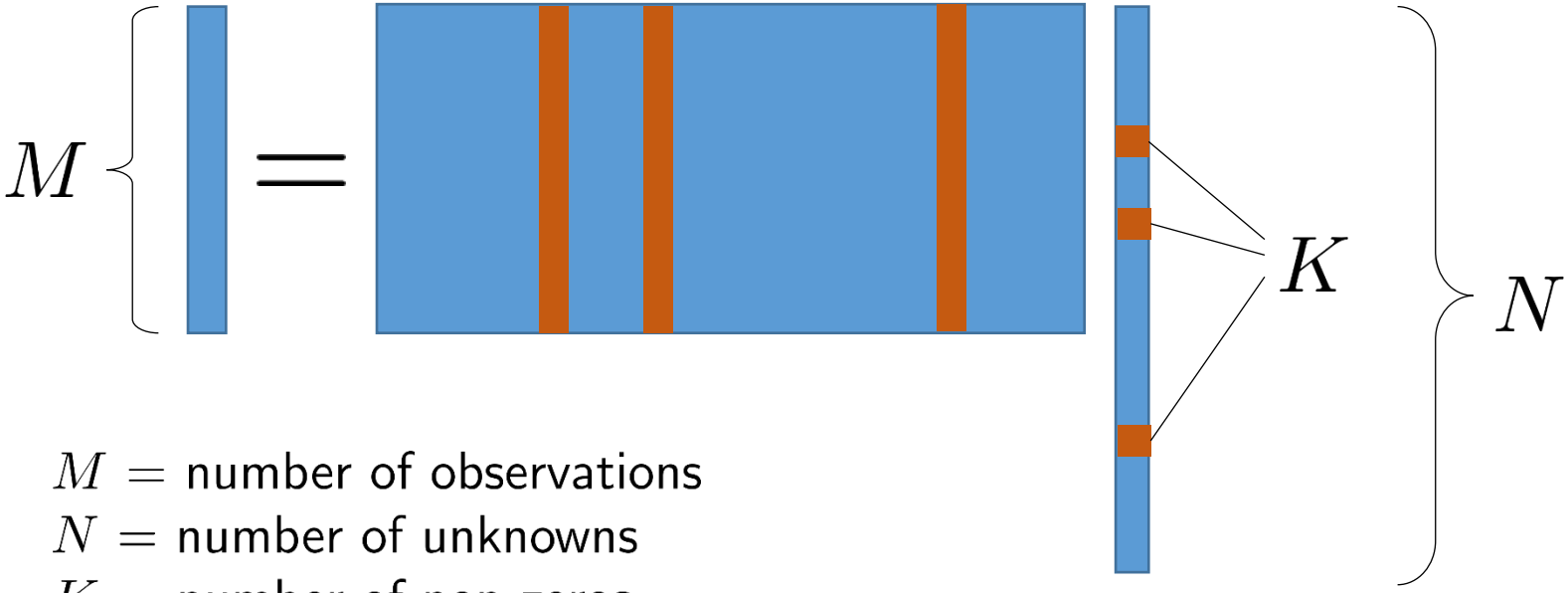
This is an over-determined system, with unique solution.

(assume columns are independent)

What is this course about?

How to solve the problem when you do **NOT** know **where** the zeros are.

Three parameters:

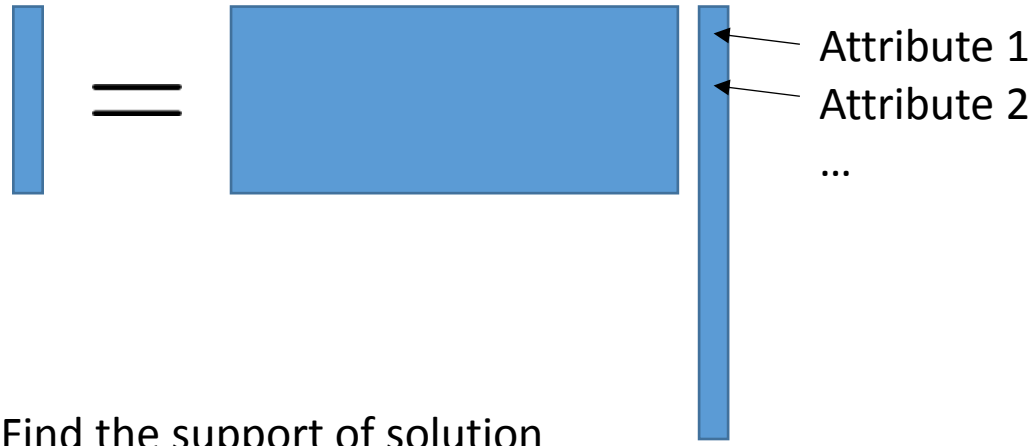


M = number of observations
N = number of unknowns
K = number of non-zeros

Under WHAT conditions can we get unique solution? and How?

Applications

Statistical Regression



Model Selection: Find the support of solution

LASSO:

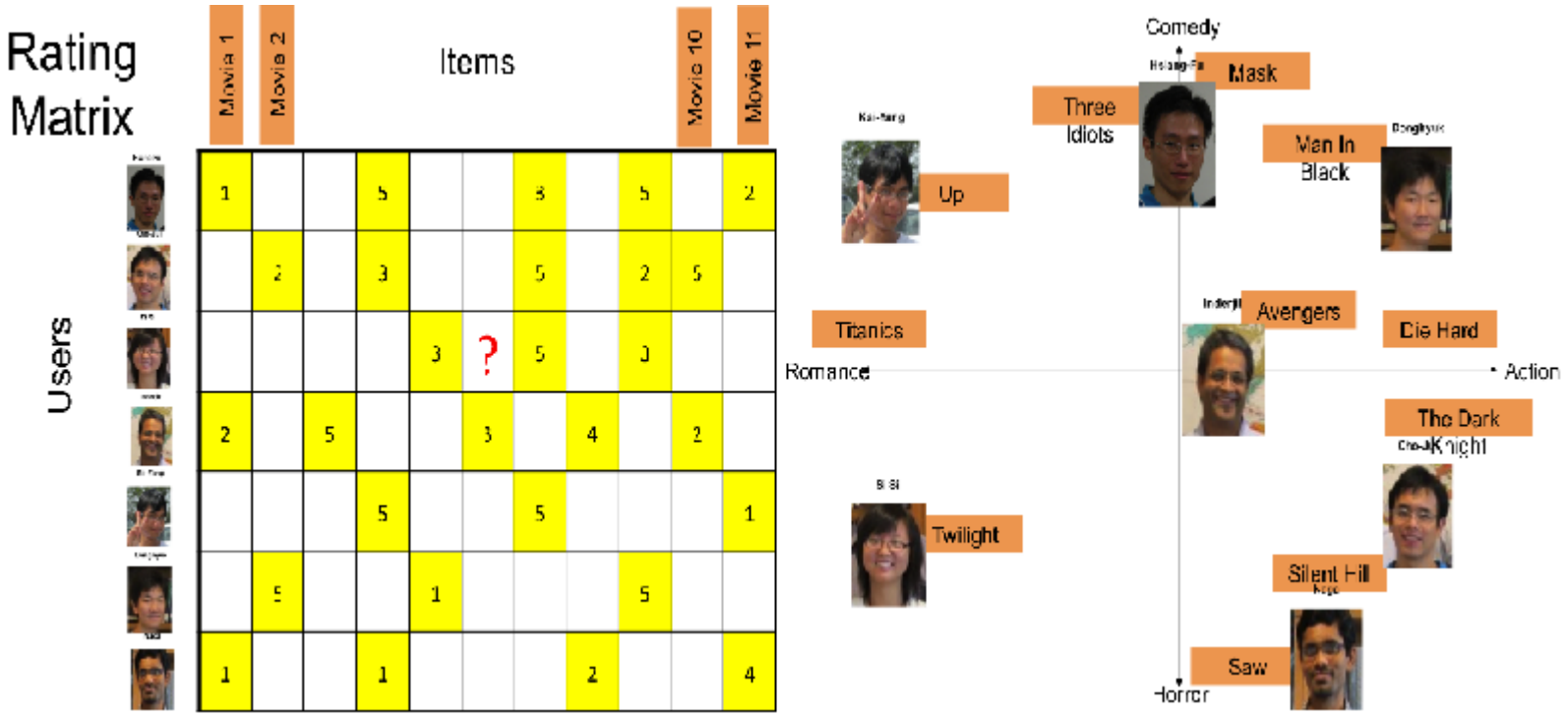
$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|^2 \quad \text{subject to} \quad \|\boldsymbol{x}\|_1 \leq \tau$$

BP-Denoise:

$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \lambda\|\boldsymbol{x}\|_1 + \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|^2$$

(for appropriate choice of \lambda and \tau, LASSO = BPDN)

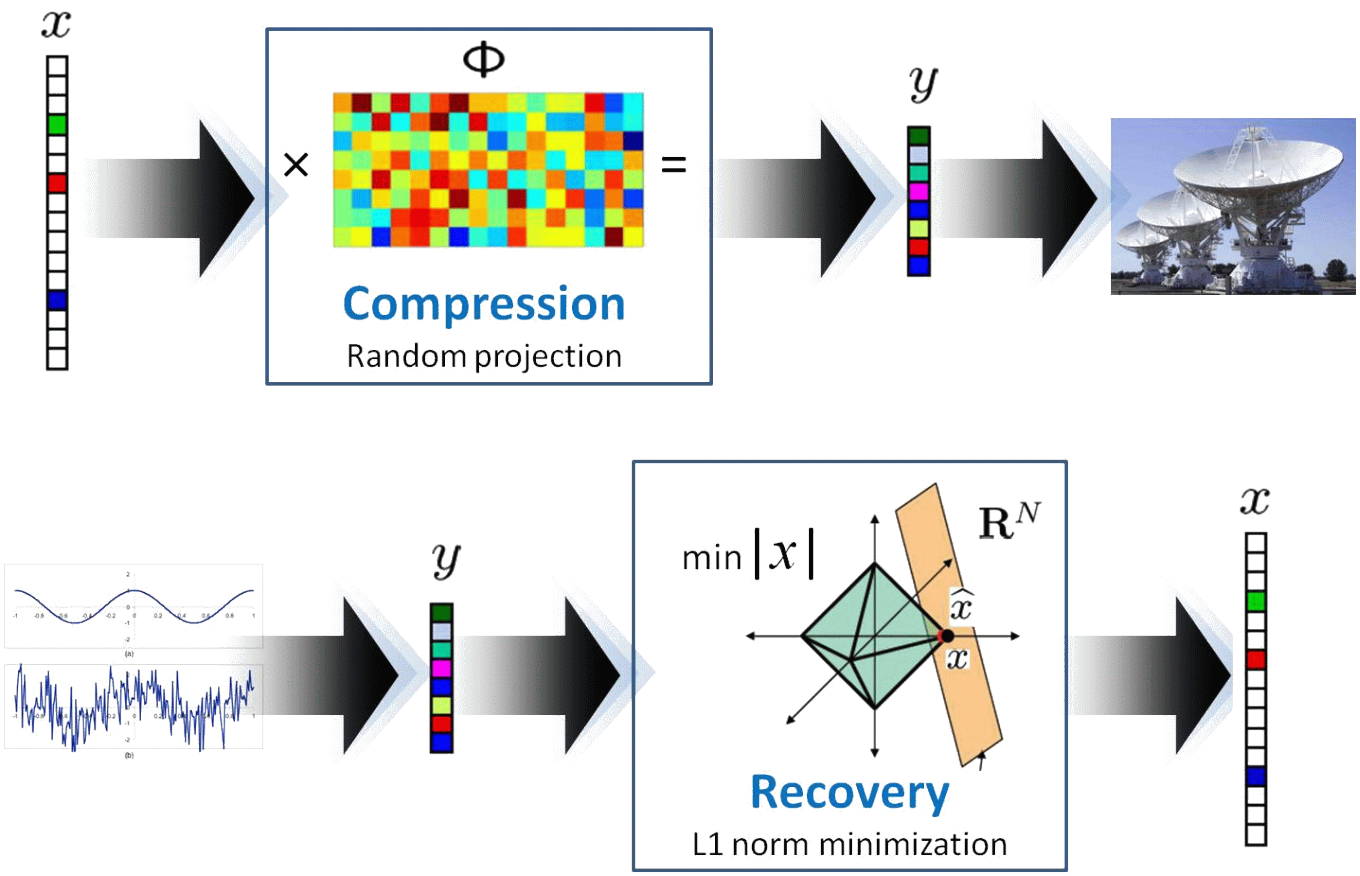
Matrix Completion



$$\underset{\boldsymbol{X}}{\text{minimize}} \quad \|\boldsymbol{X}\|_* \quad \text{subject to} \quad A(\boldsymbol{X}) = \boldsymbol{Y}$$

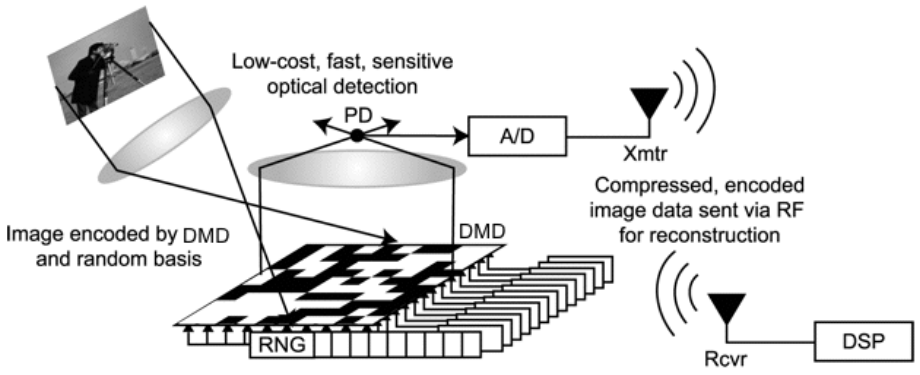
$$\|\boldsymbol{X}\|_* \stackrel{\text{def}}{=} \sum_{n=1}^{N} \sigma_n(\boldsymbol{X})$$

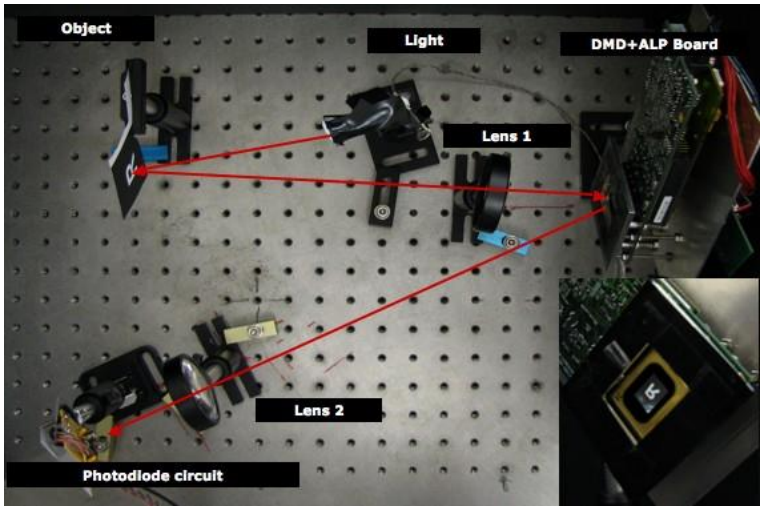
Communication System



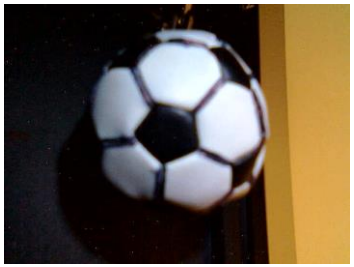
https://www.ti.rwth-aachen.de/research/applications/cs.php

Single-pixel Camera





(Courtesy of Rice University)



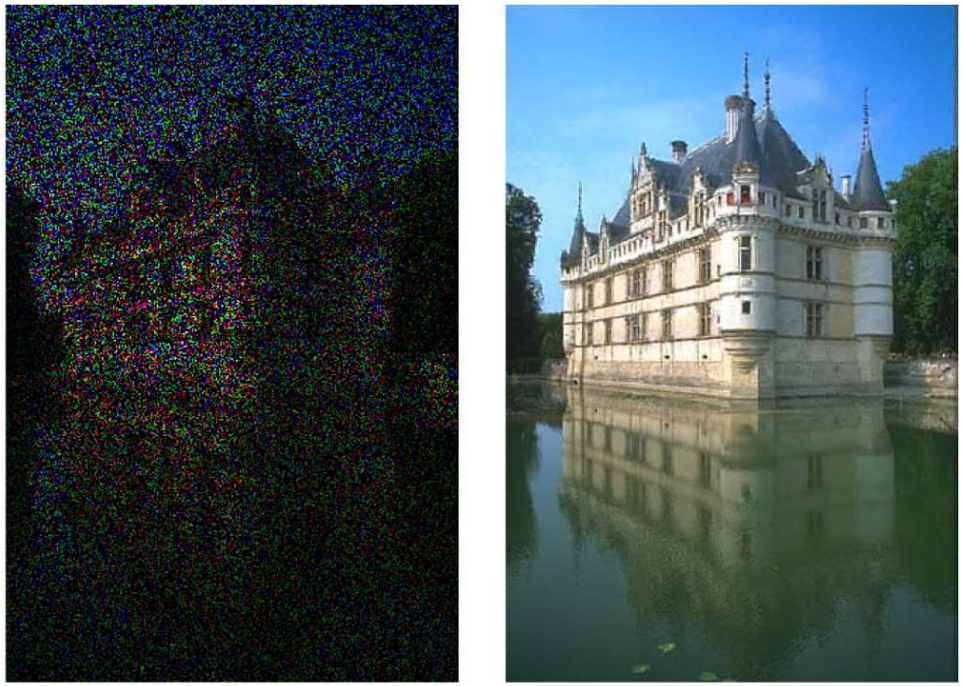
40%

40%

$$y = Ax$$

A = binary random matrix

Image Denoising



Mairal, Elad, Sapiro, T-IP 2008

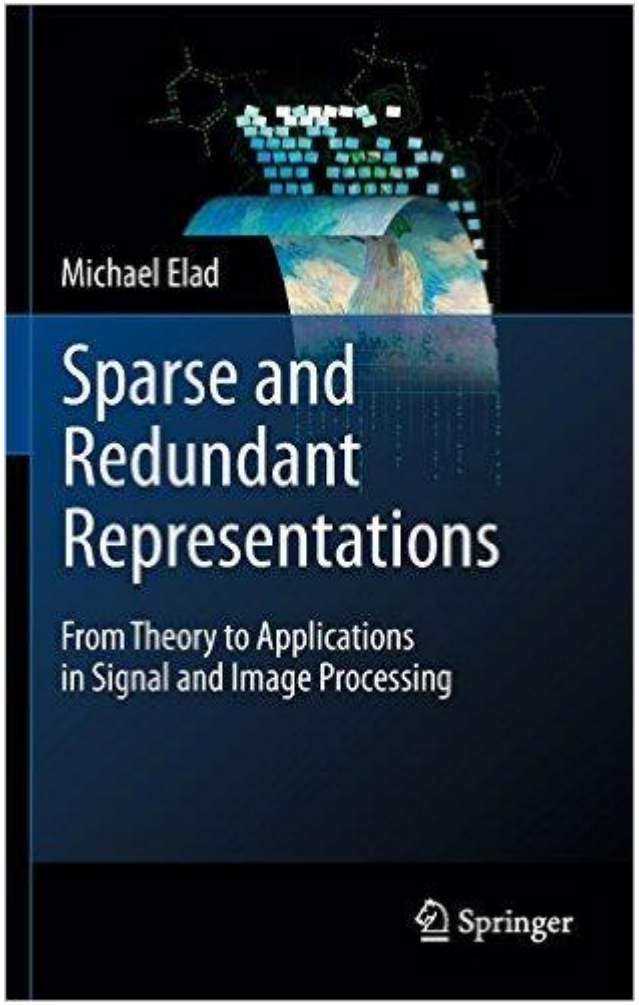
Also for image deblurring, super-resolution, etc.

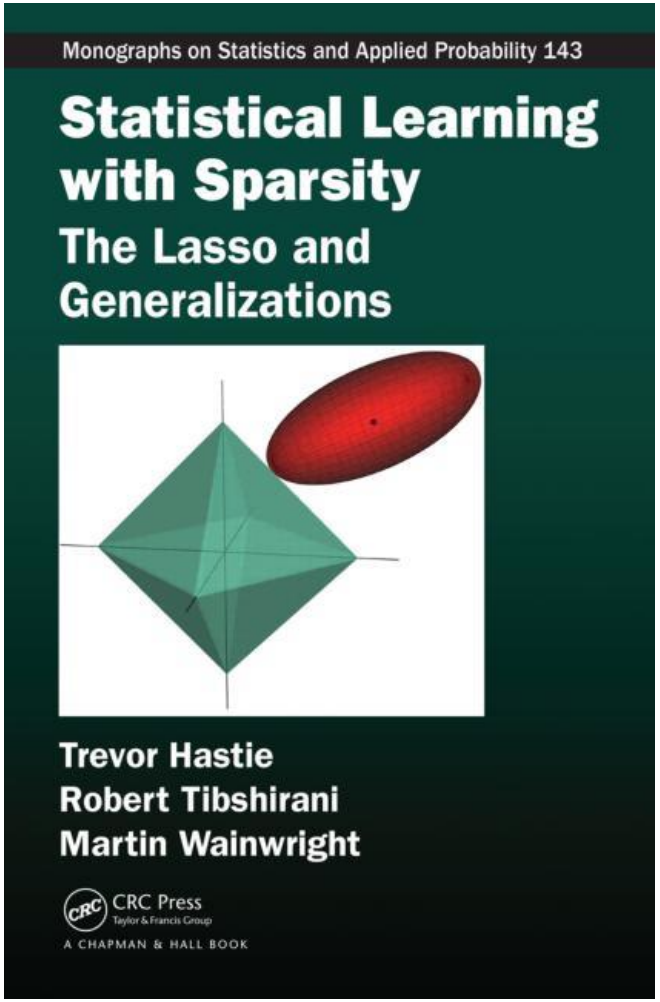
$$\underset{\boldsymbol{x}}{\text{minimize}} \quad \lambda \|\boldsymbol{x}\|_1 + \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{y}\|^2$$

Course Information

Course Website:

https://engineering.purdue.edu/ChanGroup/ECE695.html



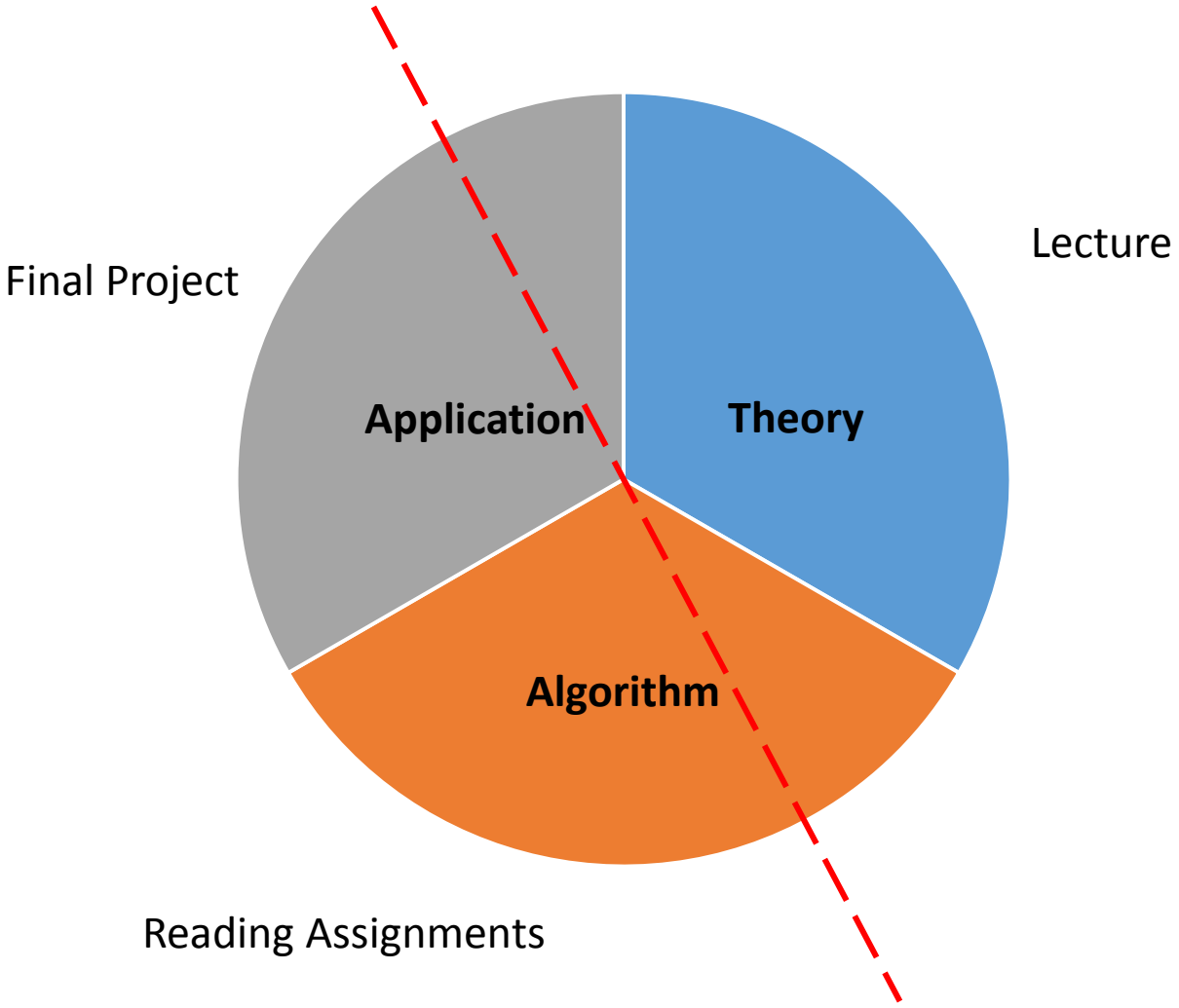


Disclaimer:

As of Jan 8, 2015,
I am still reading chapter 3 of Hastie's book.

(There are 11 chapters of the book)

My Plan: Let's learn together



Course Objective

By the end of the semester, I hope you will

- Have a basic understanding of **terminologies** and **concepts** behind sparse modeling
- Be able to comment on sparse **algorithms** and do some **implementation**
- Be able to **read** some papers in the field
- Be able to **apply** some techniques to research problems

Course Work

1. Attendance (10%)

… because

- You need to learn something
- I don’t want to talk to air

- No attendance sheet because, generally, I know who’s here and who’s not here

2. Homework (25%)

- 10 papers to read
- All very popular papers
- Write me a 2-page summary
- Tell me the problem, the algorithm, and assumptions
- Tell me your comment. Pros and cons of the method
- I will test you in the mid term

3. Mid Term (25%)

- Some basic questions about the lecture
- Some basic questions about your reading assignment

4. Project (40%)

- Group size: Min: 1, Max: 2
- Proposal (individual submission)
- Report (individual submission)
- Presentation (depend on enrollment)

- Any topic involves sparse modeling and algorithm

Auditing

- You are welcome to audit (i.e., sit in) the class
- Basic requirement:
  - 2/3 attendance
  - No texting / no apps during class
  - Arrive on time / don’t leave early
  - Attend student lecture

Will I get a good grade?

- I have given F to graduate students

- No curve
- You are competing with yourself
- For a small class like this, **I will be able to know everyone very well**
- The more effort you spend, the better grade you will get
- Historically, so far, I have no complaint about grades.

Plagiarism

- I am serious
- If it is not your work, give proper credit
- Proposal/Report: write in your **own word even** if you work as a group (Tell me your contribution)
- If I know you plagiarized, I will take very serious actions, including:
  - Fail your course immediately
  - Report to graduate school
  - Worst case: suspend you from school
- Plagiarism is a fatal crime in academia

Emergency

- Read the emergence note

- Bad weather: If you don’t feel safe, stay home
- Civil disturbance: Stay calm, find shelter

- Sick: (Recommendation) If you are sick, stay home

- Don’t worry about “attendance” in these situations. I understand.

Final Words

- Enjoy
- Have fun

Question?