Homework 1

Fall 2015
(Due: Aug 28, 2015)

The first three questions of this homework is a review of your calculus class. If you have difficulty, you should see the Teaching Assistant immediately because these materials are frequently used in ECE 302. The last four questions of this homework covers some basic ideas of set theory. They should be manageable at your level.

Homework is due on Aug 28, 2015 (Friday) at 12:00 noon. Please put your homework in the dropbox located at MSEE 268. No late homework will be accepted.

Exercise 1. (Basic, 15 points)

(a) Show that $1 + r + r^2 + \ldots + r^n = \frac{1-r^{n+1}}{1-r}$ for any $0 < r < 1$. Hence, evaluate $\sum_{k=0}^{\infty} r^k$.

(b) Using the result of (a), evaluate $1 + 2r + 3r^2 + \ldots$

(c) Using the result of (a) and (b), evaluate the sums

$$\sum_{k=0}^{\infty} k \left(\frac{1}{3}\right)^{k+1}, \quad \text{and} \quad \sum_{k=2}^{\infty} \frac{1}{k-1}$$

Exercise 2. (Basic, 10 points)

Recall that

$$\sum_{k=0}^{\infty} \frac{\lambda^k}{k!} = e^\lambda.$$

Evaluate the sums

$$\sum_{k=0}^{\infty} k \cdot \frac{\lambda^k e^{-\lambda}}{k!}, \quad \text{and} \quad \sum_{k=0}^{\infty} k^2 \cdot \frac{\lambda^k e^{-\lambda}}{k!}.$$

Exercise 3. (Basic, 15 points)

Evaluate the following integrals

(a)

$$\int_{a}^{b} \frac{1}{b-a} \left( x - \frac{a+b}{2} \right)^2 dx.$$

(b)

$$\int_{0}^{\infty} \lambda x e^{-\lambda x} dx.$$

(c)

$$\int_{-\infty}^{\infty} \frac{\lambda x}{2} e^{-\lambda |x|} dx.$$
Exercise 4. (Basic, 10 points)

(a) There are 8 processors on a computer. A computer job scheduler chooses one processor randomly. What is the sample space \( \Omega \)? If the computer job scheduler can choose two processors at once, what is the sample space then?

(b) The “ping” command is used to measure round-trip times for Internet packets. What is the sample space to model all possible round-trip times. What is the event that a round-trip time is between 10ms and 20ms?

(c) A cell phone tower has a circular average coverage area of radius 10km. We observe the source locations of calls received by the tower. What is the sample space of all possible source locations?

(d) Using your sample space from (c), what is the event that the source location of a call is between 2 and 5km from the tower?

Exercise 5. (Basic, 20 points)
Simplify the following sets

(a) \([1,4] \cap ([0,2] \cup [3,5])\)

(b) \(([0,1] \cup [2,3])^c\)

(c) \(\bigcap_{n=1}^{\infty} (-1/n,1/n)\)

(d) \(\bigcup_{n=1}^{\infty} [5,8-(2n)^{-1}]\)

Exercise 6. (Basic, 15 points)
A collection of letters, a-z, is mixed in a jar. Two letters are drawn at random, one after the other. What is the probability of drawing a vowel (a,e,i,o,u) and a consonant in either order? What is the sample space?

Exercise 7. (Basic, 15 points)
Write a MATLAB program to simulate experiments of dice.

(a) Let \(n = 100\). Generate a sequence \(X_1, \ldots, X_{100}\) such that each \(X_i\) (for \(i = 1, \ldots, 100\)) is a random integer drawn from the set \(\{1, \ldots, 6\}\). Do not use a for loop. Plot the histogram of \(X_1, \ldots, X_{100}\), with bins centered at \(\{1, \ldots, 6\}\).

(b) Repeat (a) for \(n = 10000\). Plot the histogram of \(X_1, \ldots, X_{10000}\).

(c) Let \(Y_1, \ldots, Y_{10000}\) be another random integer sequence drawn from \(\{1, \ldots, 6\}\). Let \(Z_i = X_i + Y_i\). Determine the sample space of \(Z_i\), and plot the histogram of \(Z_1, \ldots, Z_{10000}\), with bins centered at the elements of the sample space. Do not use a for loop.

(d) For \(n = 10000\), determine the probability that \(4 < Z_i \leq 7\) using the histogram.

(e) In (c), we added two different random variables \(X\) and \(Y\) to create a new random variable \(Z\). What if we add more random variables, where each is still drawn from \(\{1, \ldots, 6\}\)? Let \(K\) be the number of variables you add. Plot the histogram for the case \(K = 10\) using the command \texttt{hist(Z,K;6*K)}. You may use a for loop here. Repeat the experiment for \(K = 100\). Is it converging to anything?