Video 9

Fall 2017

Exercise 1.
Let X and Y be two independent random variables with densities

$$f_X(x) = \begin{cases} xe^{-x}, & x \geq 0, \\ 0, & x < 0, \end{cases} \quad \text{and} \quad f_Y(y) = \begin{cases} ye^{-y}, & y \geq 0, \\ 0, & y < 0. \end{cases}$$

Find the PDF of $Z = X + Y$.

Exercise 2.
Two independent random variables X and Y have PDFs

$$f_X(x) = \begin{cases} e^{-x}, & x \geq 0, \\ 0, & x < 0, \end{cases} \qquad f_Y(y) = \begin{cases} 0, & y > 0, \\ e^{y}, & y \leq 0. \end{cases}$$

Find the PDF of $Z = X - Y$.

Exercise 3.
Let X, Y, Z be three independent random variables

$$X \sim \text{Bernoulli}(p), \qquad Y \sim \text{Exponential}(\alpha), \qquad Z \sim \text{Poisson}(\lambda)$$

Find the moment generating function for the following random variables.

(a) $U = Y + Z$

(b) $U = 2Z + 3$

(c) $U = XY$

(d) $U = 2XY + (1 - X)Z$

Exercise 4.
Two random variables X and Y have the joint PMF

$$\mathbb{P}(X = n, Y = m) = \frac{\lambda_1^{n+m}\lambda_2^m}{(n+m)!m!}e^{-(\lambda_1+\lambda_2)}, \qquad m = 0, 1, 2, \ldots, \;\; n \geq -m.$$

Let $Z = X + Y$. Find the moment generating function $M_Z(s)$. Hence, find the PMF of Z.

Exercise 5.
Let $X_0, X_1, \ldots$ be a sequence of independent random variables with PDF

$$f_{X_k}(x) = \frac{a_k}{\pi(a_k^2 + x^2)}, \qquad a_k = \frac{1}{2^{k+1}},$$

for $k = 0, 1, \ldots$. Find the PDF of Y, where

$$Y = \sum_{k=0}^{\infty} X_k.$$

Hint: You may find characteristic function useful.