Video 7

Fall 2017

Exercise 1.
Alex and Bob each flips a fair coin twice. Denote "1" as head, and "0" as tail. Let X be the maximum of the two numbers Alex gets, and let Y be the minimum of the two numbers Bob gets.

(a) Find and sketch the joint PMF $p_{X,Y}(x, y)$.

(b) Find the marginal PMF $p_X(x)$ and $p_Y(y)$.

(c) Find the conditional PMF $P_{X|Y}(x \mid y)$. Does $P_{X|Y}(x \mid y) = P_X(x)$? Why?

Exercise 2.
Find the marginal CDFs $F_X(x)$ and $F_Y(y)$ and determine whether or not X and Y are independent, if

(a)

$$F_{XY}(x,y) = \begin{cases} x - 1 - \frac{e^{-y} - e^{-xy}}{y}, & \text{if } 1 \le x \le 2, y \ge 0 \\ 1 - \frac{e^{-y} - e^{-2y}}{y}, & \text{if } x > 2, y \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

(b)

$$F_{XY}(x,y) = \begin{cases} \frac{2}{7}(1 - e^{-2y}), & \text{if } 2 \le x < 3, y \ge 0, \\ \frac{7 - 2e^{-2y} - 5e^{-3y}}{7}, & \text{if } x \ge 3, y \ge 0, \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 3.
Let X, Y be two random variables with joint CDF

$$F_{X,Y}(x,y) = \frac{y + e^{-x(y+1)}}{y+1}.$$

Show that

$$\frac{\partial^2}{\partial x \partial y} F_{X,Y}(x,y) = \frac{\partial^2}{\partial y \partial x} F_{X,Y}(x,y).$$

What is the implication of this result?

Exercise 4.

(a) Find the marginal PDF $f_X(x)$ if

$$f_{XY}(x,y) = \frac{\exp\{-|y-x| - x^2/2\}}{2\sqrt{2\pi}}$$

(b) Find the marginal PDF $f_Y(y)$ if

$$f_{XY}(x,y) = \frac{4e^{-(x-y)^2/2}}{y^2\sqrt{2\pi}}$$

Exercise 5.

Let X and Y be two random variables with joint PDF

$$f_{X,Y}(x,y) = \frac{1}{2\pi} e^{-\frac{1}{2}(x^2+y^2)}.$$

(a) Find the PDF of $Z = \max(X,Y)$.

(b) Find the PDF of $Z = \min(X,Y)$.

You may leave your answers in terms of the $\Phi(\cdot)$ function.