

Video 4

Fall 2017

Exercise 1.

Consider an information source that produces numbers k in the set $S_X = \{1, 2, 3, 4\}$. Find and plot the PMF in the following cases:

- (a) $p_k = p_1/k$, for $k = 1, 2, 3, 4$. Hint: Find p_1 .
- (b) $p_{k+1} = p_k/2$ for $k = 1, 2, 3$.
- (c) $p_{k+1} = p_k/2^k$ for $k = 1, 2, 3$.
- (d) Can the random variables in part (a)-(c) be extended to take on values in the set $\{1, 2, \dots\}$? If yes, why? If no, why not? Hint: You may use the fact that the series $1 + \frac{1}{2} + \frac{1}{3} + \dots$ diverges.

Exercise 2.

Let

$$g(X) = \begin{cases} 1, & \text{if } X > 10 \\ 0, & \text{otherwise.} \end{cases} \quad \text{and} \quad h(X) = \begin{cases} X - 10, & \text{if } X - 10 > 0 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find $\mathbb{E}[g(X)]$ for X as in Problem 1(a) with $S_X = \{1, \dots, 15\}$.
- (b) Find $\mathbb{E}[h(X)]$ for X as in Problem 1(b) with $S_X = \{1, \dots, 15\}$.

Exercise 3.

Two dice are tossed. Let X be the absolute difference in the number of dots facing up.

- (a) Find and plot the PMF of X .
- (b) Find the probability that $X \leq 2$.
- (c) Find $\mathbb{E}[X]$ and $\text{Var}[X]$.

Exercise 4.

Let X be a random variable with PMF $p_k = c/2^k$ for $k = 1, 2, \dots$

- (a) Determine the value of c .
- (b) Find $\mathbb{P}(X > 4)$ and $\mathbb{P}(6 \leq X \leq 8)$.
- (c) Find $\mathbb{E}[X]$ and $\text{Var}[X]$.

Exercise 5.

A modem transmits a +2 voltage signal into a channel. The channel adds to this signal a noise term that is drawn from the set $\{0, -1, -2, -3\}$ with respective probabilities $\{4/10, 3/10, 2/10, 1/10\}$.

- (a) Find the PMF of the output Y of the channel.
- (b) What is the probability that the output of the channel is equal to the input of the channel?

- (c) What is the probability that the output of the channel is positive?
- (d) Find the expected value and variance of Y .