Video 11

Fall 2017

Exercise 1.
Given the functions $a(t)$, $b(t)$ and $c(t)$, let

$$\begin{aligned} g(t,1) &= a(t) \\ g(t,2) &= b(t) \\ g(t,3) &= c(t). \end{aligned}$$

Let $X(t) = g(t, Z)$, where Z is a discrete random variable with PMF $\mathbb{P}[Z = 1] = p_1$, $\mathbb{P}[Z = 2] = p_2$, $\mathbb{P}[Z = 3] = p_3$. Find, in terms of the p_1, p_2, p_3, $a(t)$, $b(t)$ and $c(t)$,

(a) $m_X(t)$

(b) $R_X(t_1, t_2)$

Exercise 2.

(a) Show that $C_X(t_1, t_2) = R_X(t_1, t_2) - m_X(t_1)m_X(t_2)$.

(b) Show that $C_{XY}(t_1, t_2) = R_{XY}(t_1, t_2) - m_X(t_1)m_Y(t_2)$.

Exercise 3.
Consider the random process

$$X(t) = A\cos(t) + (B + 1)\sin(t),$$

where A and B are two independent random variables with $\mathbb{E}[A] = \mathbb{E}[B] = 0$, and $\mathbb{E}[A^2] = \mathbb{E}[B^2] = 1$.

(a) Find $m_X(t)$

(b) Find $R_X(t_1, t_2)$

(c) Find $C_X(t_1, t_2)$

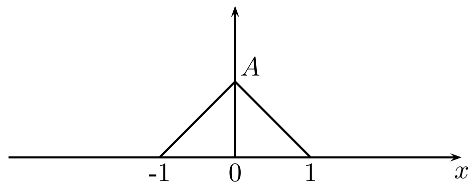
Exercise 4.
Let $X[n]$ be a discrete-time random process with mean function $m_X[n] = \mathbb{E}\{X[n]\}$ and correlation function $R_X[n, m] = \mathbb{E}\{X[n]X[m]\}$. Suppose that

$$Y[n] = \sum_{i=-\infty}^{\infty} h[n - i]X[i]. \quad (1)$$

(a) Find $m_Y[n]$

(b) Find $R_{XY}[n, m]$

Exercise 5.
Let $g(x)$ denote the triangular function shown below.



(a) Find the power spectral density $S_X(\omega)$ if $R_X(\tau) = g(\tau/T)$.

(b) Find the autocorrelation function $R_X(\tau)$ if $S_X(\omega) = g(\omega/W)$.

(c) Let $R_Y(\tau) = R_X(\tau)\cos(\omega_0\tau)$. Find and sketch $S_Y(\omega)$.

Exercise 6.
Let $Y(t) = X(t) - X(t+d)$.

(a) Find $R_{X,Y}(\tau)$ and $S_{X,Y}(\omega)$.

(b) Find $R_Y(\tau)$.

(c) Find $S_Y(\omega)$.

Exercise 7.
Let $X(t)$ be a zero-mean WSS process with autocorrelation function $R_X(\tau)$. Let $Y(t) = X(t)\cos(\omega t + \Theta)$, where $\Theta \sim \text{uniform}(-\pi, \pi)$ and Θ is independent of the process $X(t)$.

(a) Find the autocorrelation function $R_Y(\tau)$.

(b) Find the cross-correlation function of $X(t)$ and $Y(t)$.

(c) Is $Y(t)$ WSS? Why?